Faculty of Science

Prospectus 2008 - 2009

Mathematics

Master

Radboud University Nijmegen

Preface

This booklet is the propectus for the Master program of Mathematics at the Radboud University Nijmegen. It contains information about the objectives, the goals and the contents of the program.

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This prospectus has been made with great care. However the authors are not responsible for inaccuracies. If you have comments or proposals for improvements don't hesitate to contact them.

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1 Introduction

1.1 Structure

The Radboud University Nijmegen offers a Master of Science program in Mathematics. The English taught MSc program takes two years and provides students with a thorough knowledge of the relevant mathematics. The first year consists of core courses and electives in mathematics. The second year is largely devoted to the final thesis work, which involves participating in one of the department's advanced research projects or a traineeship or an internship within a company. If you pass the MSc exam you are awarded 'Master of Science'.

The Radboud University Nijmegen is a general university, offering almost all possible academic Programs, ranging from Arts and Law, to Medicine and Science. This Master program allows a substantial choice of topics from all these areas, thereby offering the possibility to combine Mathematics with other studies.

1.2 Admittance

In possession of a bachelor degree in Mathematics, one can of course enter the Mathematics Masterfase. In fact, a bachelor degree in Mathematics from any Dutch university will suffice. With an equivalent degree one can also be admitted, but approval of the Exam Committee Mathematics will be required. In this case, the Exam Committee can demand that one adds specific courses to the master program, depending on the Master specialization.
Eg: With a bachelor degree in Physics with a minor in Mathematics, one will be admitted to the Mathematics Master with the specialization Physics.

1.3 Credit point system

The Radboud University uses the European Credit Transfer System (ec) employed by all universities in the European Union. One year consists of 60 ec credits, the total Master program consists of 120 ec.

1.4 Dutch Master Program in Mathematics

The Departments of Mathematics of the Dutch universities have coordinated their efforts to enhance their Master Programs in Mathematics. Part of the cooperation is aimed at organizing joint courses in mathematics. The joint courses offer the students the highest quality of instruction and open opportunities for interaction with students of other institutes of mathematics. For students who intend to pursue a PhD program after completion of their Master Program the joint program may widen the range options for continuing their studies. For more information concerning the Dutch Master Program in Mathematics see the web site www.mastermath.nl .

Each master student in mathematics is obliged to attend courses from the Dutch Master Program in Mathematics with a total weight of 30 ec or more.

1.5 Mathematics clusters

Mathematics clusters form a new part of the Dutch mathematical landscape. A cluster is a collaboration between a number of Dutch universities, which is organized around a specific research area. Nijmegen takes part in two of the three clusters that are currently active, viz. DIAMANT (Discrete, Interactive & Algorithmic Mathematics, Algebra & Number Theory - involving CWI, RU, TU/e, UL) and GQT (Geometry and Quantum Theory, consisting of RU, UvA, UU). The RU does not take part in the third cluster NDNS (Nonlinear Dynamics of Natural Systems: CWI, RUG, UL, UU, VU).

While primarily founded in order to boost mathematical research in the Netherlands, the clusters are of great importance to master's students. This is firstly because they organize the bulk of the courses within the Dutch Master Program in Mathematics, and secondly because in writing your master's theses you will probably be attracted by some of the research themes offered by one of the clusters that Nijmegen takes part in.

The relevant websites are:
DIAMANT: www.win.tue.nl/diamant
GQT: www.gqt.nl

1.6 Mathematical Research Institute

The Mathematical Research Institute (web site www.math.uu.nl/mri) is one of the research schools in The Netherlands. It is the combined research school for the mathematics departments of the universities of Groningen, Nijmegen, Twente and Utrecht. The MRI is one of three national research schools in mathematics in the Netherlands. The others are EIDMA and Stieltjes . The MRI participates in the European research institute EURANDOM .

The research programme of the Mathematical Research Institute focuses on the fundamental aspects of mathematics, as well as on interactions with its applications.

Each year the MRI organizes the Master Class and the Spring School. The Master Class offers a one-year programme centered around a theme that is close to one of the research areas supported by the MRI. The Spring School programme offers a 4 to 8-week intensive course (in March, April or May) in an advanced topic.

The 2008-2009 topic of the MRI Master Class is: Hodge Theory.

1.7 Master thesis

At the start of the Master program, the student is expected to contact one or more prospective Master thesis supervisors to discuss a program. Normally the student will contact a thesis supervisor in the first semester of the Master program. The courses that are needed to prepare for the Master thesis work are determined by the Master thesis supervisor and the student together. It is therefore advisable to contact the prospective Master thesis supervisor to discuss the content of these courses. The individual program needs approval by the examination committee. To select a prospective Master thesis supervisor, please look at the descriptions of the different specialisations within the department of mathematics. The contact persons of the departments can be approached at any stage for information or to set up a program in the electives section.

2 Program

2.1 Variants

The Master program at the Faculty of Science is offered in four variants: a research (O) variant, a communication (C) variant, an education (E) variant, and a business and management (MA) variant. At this moment, only the research variant has a complete program in the English language. The other variants are primarily aimed at the Dutch market and the Dutch educational system, and are therefore taught in Dutch.

- The O-variant leads students to a high level of knowledge in mathematics. It consists of advanced courses and a final research project including a Master thesis and an oral presentation of it. Students with this MSc in Mathematics are admissible to a PhD program. The program is suited as preparation for an academic career, in particular via a subsequent PhD study, but also for a career as mathematical researcher outside the universities.
- The C-variant is intended for a job in science communication in a broad sense. The program prepares students for a career in popularisation of science.
- The E-variant is intended as a preparation for a job in teaching mathematics and mathematics curriculum development.
- The MA-variant is intended as a preparation for jobs in the field of management. It prepares students for a career in science-related business and administration and for innovation and enterprise from a mathematical perspective.

In each variant the student chooses a specialisation within mathematics. The specialisation consists of a research project in one of the mathematics research groups of the Faculty of Science, including a Master thesis and an oral presentation and 30 ec of advanced courses to be selected in correspondence with the research topic.

2.2 Regulations Graduation project

The Graduation project is the completion to the Master Program. The central part consists of either a thesis or an apprenticeship. In both cases there is a staff member to supervise. In this paragraph one can find a more elaborate description of the Graduation Project.

Graduation Project including master thesis

The project consists of:

1. Acquiring knowledge about a specific subject by way of literature study, consultation and/or participating in a seminar.
2. Conducting scientific research.
3. Writing a thesis.
4. Presenting the master thesis and defending it in front of an audience of experts.

Graduation Project including an apprenticeship

The project consists of:

1. Acquiring knowledge about a specific subject by way of literature study, consultation and/or participating in a seminar
2. Doing work experience during an apprenticeship.
3. Writing an apprenticeship report
4. Presenting the apprenticeship report and defending it in front of an audience of experts.

Assessment

The procedure for graduation project assessment consists of the following steps:

1. The supervisor approves of the thesis and /or the apprenticeship report, and notifies the program coordinator.
2. The program coordinator appoints a second rater, the supervisor organizes the presentation of the master thesis or the apprenticeship report in consultation with the student and the second rater.
3. After the presentation the supervisor determines the grade for the graduation project in consultation with the second rater.

2.3 The research variant (O-variant)

The program for the O-variant can be found in the table below. The amount of ec points is given between parentheses.

O-variant	
O-package	(30)
Mathematical Electives	(17)
Electives	(24)
Master Thesis Project	(40)
Philosophy	(3)
Free Electives	(6)

2.4 The communication variant (C-variant)

The communication variant (C) educates people for the areas of scientific communication (research, applications, media). The student who graduates in Communication is more than a beta (science) student, and has acquired complementary theoretical insights and communicative skills that broaden his own field of expertise (beta-gamma-integration). He has an insight in communication about innovation processes and processes of change, as well as an insight in the use of (mass) media and popularization.

If you are interested in the interaction between science and the society, science communication might be an interesting way to go. Science communication is one of the graduation variants on the beta-faculty in Nijmegen. Among other things, it deals with perceptions, participation processes, knowledge production, interdisciplinarity and risks and uncertainties in science. Moreover, much attention is paid to writing skills (essay, columns), presentation skills and research methods. During your graduation project (30 ECTS), you link up theory from the courses with your beta background.

The job profile entails three fields: intermediary organisations between science and society (advisory bodies, interest groups and gouvernements), science communication research and science journalism. The Science Communication graduation variant is not only a very interesting new field of study for which there is a need on the labour market, it provides you with knowledge that may come in handy in every speciality!

More information can be obtained at www.filosofie.science.ru.nl/ from: Prof. dr H. Zwart, Room: HG 02.808, Tel. 3652038, E-mail: h.zwart@science.ru.nl

Masterfase

In this main subject in the Masterfase 63 ec are contributed to the beta education, and 57 ec to the modules and apprenticeship of the C-variant.

C-variant	
Mathematical Specialisation	(30)
Mathematical Electives	(24)
C-package	(27)
Apprenticeship & Master Thesis	(30)
Philosophy	(3)
Free Electives	(6)

The C-package

The C-package includes: Framing Knowledge (3), Knowledge Society (3), Science & Media: strategies and trends (only in Dutch) (3), Introduction Science Communication (3), Science & Societal interaction (3), Risk Communication (3), Boundary-Work: The Tension between Diversity and Sustainability (3), Visible Scientists (3), Zichtbare en onzichtbare wetenschappers (3).

Free Electives

The free elective space can be filled with courses lectured by the Communication faculty. The approval for other free electives (for instance from Communication sciences, Psychology, Sociology, Biomedical Sciences, or other universities) will be given in consultancy with the student.

Apprenticeship & Master thesis. (24 ec + 6 ec)

The apprenticeship (24 ec) is communication research regarding the interface between Science and Society. Topics can be for instance: the role of scientists (experts) in conflicts between interest groups, conceptualization of social and scientific topics in the mass media (internet, film, tv. etc.), risk communication concerning disasters, the making of a Strategical Communication plan, the effectiveness of newsletters and websites, and political and social opinion forming.

The topic and methodology are always determined in consultation with the student, depending on his wishes and foreknowledge. The apprenticeship can be placed external (within a nonprofit organization, ministry, or commercial organization), or accommodated on the faculty. The master thesis (6 ec) considers the results from the communication research in context with relevant literature, and makes recommendations for further research.

The quality and intensity if the supervision regarding the student will be guaranteed through apprenticeship agreements made with the internal and external supervisors.

2.5 The educational variant (E-variant)

E-variant	
Mathematical Specialisation	(30)
Mathematical Electives	(14)
E-traineeships	(57)
Master Thesis Project	(10)
Philosophy	(3)
Free Electives	(6)

The educational variant has a major course of 30 ec, in with preferably subjects are integrated that make it possible to view school math from a higher point of view, for instance because of the historical importance of the development of Mathematics.(the Master specialization Mathematics and education). Furthermore 14 ec free electives math, obligatory 3 ec Philosophy and 6 ec free electives. The course introduction to math didactics (8 ec), with is linked to supervising pupils in the project Mathematical Thinking Processes, can be taken in the fourth year, and can be included in the career preparation. By then, the study will be wound up in the fifth year with a final report of the career preparation at the ILS and a didactic thesis (10ec), if possible, both in mutual coherence.

2.6 The Management and Applications variant (MA-Variant)

MA-variant	
Mathematical Specialisation	(30)
Mathematical Electives	(24)
MA-package	(30)
Master Thesis Project	(27)
Philosophy	(3)
Free Electives	(6)

Goal

The set-up and filling-in of the management and applications variant strives to integration of beta and gamma aspects. In the MA variant the appliance of beta knowledge is the central aspect. Students are required to be capable of ding beta research in an applied setting; that also includes the handling of the company and managements difficulties, which cohere with those appliances.

For the filling-in of the MA variant thoughts go out to a mutual consensus about the contents of scientific professional education (in particular research apprenticeships) in the fourth year and the external apprenticeship location (organization that applies such beta know-how) in the fifth year.

Master education

In the main part of the Master 63 ec are contributed to the beta education, and 57 ec to the modules and apprenticeship of the MA-variant.

Courses

This year the following courses are being offered:

- *MA- courses* business and society (5ec), Organization knowledge (ec).
- *MA-integration courses:* Innovation management (5ec), Strategy and Marketing (5 ec), Financial Economic Management (5ec).
- *MA-free electives (*5ec*, choices of)*: Research Strategy and Management (3ec), Knowledge and entrepreneurship (3ec), Industrial Fine Chemistry, General Management skills.

Master project

The master project consists of conducting research in the cutting edge of science, technology, society and organization, mostly within a profit- or non-profit organization or the government.

Together with the student projects and research questions will be actively looked for, in which not only company or management knowledge is required, but also a science/ physics background is an advantage. Also it is desirable that a connection can be made between the fourth' year beta research and the master project MA. We emphasize the expression master project and not apprenticeship, because the student isn't supposed to just work at a company or institution, but, based on a completed phrasing of a question, handles a problem that is relevant to the organization.

Possible research areas for master projects are:

- Developing and/or evaluating of instruments for innovation-, science-, and environment policy.
- Researching the bottlenecks in the implementation of a technological innovation in an organization.
- Researching the consequences of actions of (for instance) environmentalists on company policy.
- Developing and/or evaluating instruments for personnel policy of a R&D department.
- Evaluation of the decision process of research projects in a R&D department.
- Developing instruments for improving the collaboration between universities and businesses.
- Developing instruments for supporting produce development.

3 Specialisations

3.1 Master specialisation

Mathematical research in the Radboud University Nijmegen is focused on the following themes:

- Algebra and Logic
- Mathematical Physics
- Mathematics and Education

In any variant (O, C, E, MA) it is important to find a specialisation for graduation and, accordingly, a matching program of courses. Electives can be chosen freely, but the total package has to be approved by the examination board.

3.2 Algebra and Logic

Algebra, originally the study of arithmetic and the solution of equations, transformed over the latter part of the 19th century and the first part of the 20th into the study of abstract algebraic structures and their relationships. This transformation sprang from work on various types of algebraic systems including systems of linear equations, permutation groups, and propositional logics. This 'modern algebra' has had a profound impact on many domains of mathematics including functional analysis, geometry, topology, and logic. As a consequence algebra has also been deeply influenced by various other fields. Applications in mathematical physics focus on operator algebra, algebraic geometry, and symmetry groups among other. Another source of applications and inspiration for algebra is logic. The algebraic nature of logic may be seen as having been explored already by Leibniz but was firmly identified by Boole in his analysis of 'the laws of thought' in the 1850s. These insights were properly turned into modern algebra by Tarski and Lindenbaum's construction of the free algebra describing a logic. The connection between algebra and logic has also influenced algebra focusing on lattice-ordered algebras, categorical structures, categories as generalised algebras, algorithmic and constructivist issues, and topological duality theory as central subjects. This part of algebra has grown and expanded over the last half century, driven by the advent of computers and computer and information sciences. As the methods and issues of these new disciplines deepen and get more sophisticated, the synergistic impact on algebra is getting ever more important.

The Algebra and Logic group has a seminar:
www.math.ru.nl/~mgehrke/Algebra&Logic/Algebra&Logic.htm

The seminar provides an opportunity for students to get acquainted with the work of the group and provides a setting for informal talks by visitors to the group.

Research in Algebra and Logic

The research of the group covers a broad cross-section of the classical modern algebra topics including polynomial rings, groups, fields and lattice-ordered algebras. An identifying common thread is the focus on the interplay of computational, logical, and geometric ideas and methods. Gehrke's work focuses on topological methods in algebra both as they pertain to applications in geometry and physics and in logic and foundations of computer science.

Van den Essen's research is in Affine Algebraic Geometry, a recently named subbranch of Algebraic Geometry which focusses on the a study of affine varieties. Bosma's work is in computer algebra. His research is focussed on number theoretic algorithms and has applications in cryptography. He has also been involved in the development of the computer algebra programme, Magma. Souvignier's research area is group theory with special emphasis on algorithmic methods in computational representation theory and crystallographic groups. The leading question of Veldman's research has been to take Brouwer's proposals for a reform of mathematics seriously and to develop mathematics according to intuitionistic principles.

For more and current information on the research of the group, consult the page www.ru.nl/science/ca and the individual links from there.

Master Programme

The courses of the Master Programme in Algebra and Logic consist of the following named courses along with a varying number of special topics courses.

Lattice theory	Commutative algebra
Set theory	Polynomial mappings
Theory of recursive functions	Duality theory
Topics in computer algebra	Intuitionistic mathematics
Model theory	Groups and representations

This programme connects well with the Bachelor minor in informatics, and electives within the Master in theoretical computer science are recommended. For more information please follow the eductional programme link from the webpage of the group or talk to one of the members of the group.

3.3 Mathematical Physics

From the time of Newton (1642-1727) until about 1930, mathematics and theoretical physics were inseparable. Breakthroughs typically took place simultaneously in both areas, and progress at both fronts often even resulted from the work of a single scientist, such as Newton himself, Huygens, Euler, Lagrange, Laplace, Fourier, Gauss, Poisson, Cauchy, Jacobi, Hamilton, Riemann, and Poincaré. Their work provided the foundations of 'classical' mathematical physics (as well as of large areas of mathematics), which culminates in the field of partial differential equations (the Maxwell equations are a case in point). A second stage in the development of mathematical physics is connected with some of the greatest names in 20th century science, like Einstein, Born, Dirac and Wigner on the physics side, and Hilbert, Weyl, von Neumann and Kolmogorov on the maths side. The cross-fertilization of mathematics and physics led by these people was instrumental in establishing key areas of modern physics like general relativity and quantum mechanics as well as parts of mathematics like differential geometry, Lie groups and functional analysis.

This typical cross-fertilization subsided between about 1930-1975, when research at the frontiers of physics felt no need for advanced or new mathematics (whose relevance to physics was even openly derided by Feynman), whilst simultaneously mathematics began to be developed according to its own internal criteria established by Hilbert and others (notably the French Bourbaki group). (In addition, after 1945 some of the greatest mathematicians like

Grothendieck refused to make use of insights from modern physics because of its connection to nuclear weapons.) This has led, for example, to the creation of modern algebraic geometry and algebraic number theory by Weil, Grothendieck, Serre, Deligne, and others. This development may be said to have culminated in the extremely deep and abstract proof of Fermat's Last Theorem by Wiles in the mid-1990s. From about 1975, however, mathematical physics has began to regain the élan it used to have.

First, mathematicians like Atiyah, Singer and Penrose, and physicists like 't Hooft and Witten recognized the connection between differential geometry and gauge theories. This connection goes via the notion of index theory (originating in analysis) and is crucial, for example, in the technical implementation of Sacharov's scenario for baryogenesis mentioned earlier. This recognition has led to very important progress in both physics (magnetic monopoles, instantons, anomalies, and other topological phenomena in classical and quantum field theory) and mathematics (e.g. Donaldson theory and Floer homology). In its immediate wake, deep relationships between algebraic geometry and quantum field theory and string theory were discovered and developed by Witten, Kontsevich and others. The work of Dijkgraaf, Verlinde and Verlinde also played an important role here.

Second, Connes began to develop an entire new domain of mathematics called noncommutative geometry on the basis of ideas from quantum physics (e.g. the Dirac equation), operator algebras (an area of mathematics created by von Neumann in the 1930's) and index theory (the field launched by Atiyah and Singer just mentioned). This body of work has led to breakthroughs in a number of areas in pure mathematics (like index theory and foliation theory). Furthermore, as might have been expected, the subject was successfully applied to physics within a decade after its inception, for example to the quantum Hall effect, the theory of quasicrystals, and the Standard Model of elementary particle physics. More recently, noncommutative geometry has also been related to renormalization theory in perturbative quantum field theory, and to string theory.

Third, the classical area of integrable systems (going back to Lagrange, Jacobi and others, with important later contributions by Lax) underwent a complete rejuvenation in that it got related to the Langlands program (originally an area of pure mathematics in which number theory and representation theory interacted). Also, the notion of a Frobenius manifolds emerged from the work of Witten, Manin, Dijkgraaf and others as a new setting for integrable systems. These three areas together have culminated in an independent field of research called the geometric Langlands program.

The above developments have been widely recognized by the mathematical community. Atiyah has been awarded both of the two most prestigious prizes in mathematics, viz. the Abel prize (with Singer) and the Fields Medal. Connes and Kontsevich won the Fields Medal as well, as did Witten (although he is a physicist). Lax was awarded the Abel Prize. Penrose has won the Wolf Prize and numerous other awards, as did 't Hooft. And so on and so forth.

Research at Nijmegen

Each of the three current research directions in mathematical physics just mentioned is well represented at Nijmegen, and there are other themes as well, so students interested in research in mathematical physics are offered a rich choice. Clauwens' research is in algebraic topology. Heckman's research lies in the interaction between Lie theory, integrable systems, and geometry, and is closely related to the third topic above. It is currently centered around the link between the geometric Langlands program and Hitchin's integrable system. Landsman's research combines noncommutative geometry with quantization theory (i.e. the theory that tries to establish the precise mathematical relationship between classical and quantum physics). One goal is the quantization of singular spaces, in the hope of eventually developing a quantum theory of the Big Bang (a purely classical notion which according to Stephen Hawking and others is probably smoothened out in quantum theory). Maassen works in quantum probability, including the application of stochastic calculus to the interaction of molecules with light, and the functional analysis of quantum noise. In addition he works in the modern theory of quantum information and quantum computing. Müger specializes in category theory, operator algebras and their applications to quantum field theory. He is also interested in constructive quantum field theory. Steenbrink's work is mainly in algebraic geometry. Over the last few years it has focused on two streams: the study of discriminant complements and moduli spaces, and the study of threefolds which are double covers of projective space (double solids).

Preparation

When coming from the BSc Program in mathematics, the student is recommended to prepare for the MSc Program by filling in the free space of the BSc Program in the way described in the Bachelor Prospectus. This is as follows:

1st semester: Mechanics 1 & 2 (6 ec), Laboratory course in Physics 1a (1 ec)
2nd semester: Mechanical waves(2 ec), Introduction to quantum mechanics (3 ec)
3d semester: Analytical mechanics (3 ec), Vibrations and waves (3 ec)
4th semester: Quantum mechanics 1a & 1b (6 ec), Electricity and magnetism 1 & 2 (6 ec), Introduction Fourier theory (3 ec), Complex functions (3 ec)
6th semester: Introduction to partial differential equations (6 ec), Analytic geometry (3 ec)

Special relativity (3 ec) is highly recommended as well. Before entering the master program in Mathematical Physics, one is recommended to take Quantum mechanics 2 (5 ec) and Quantum mechanics 3 (6 ec). Otherwise, one has to do these courses during the master itself.

Master program

The following courses, to be taken during the fourth and fifth years, comprise the actual Master Program.

From mathematics
- Hilbert space and quantum mechanics (6 ec)
- One or two items chosen from:
    - Probability theory (6 ec)
    - Stochastic processes (6 ec)

From physics
- Group theory (6 ec)
- Classical electrodynamics (6 ec)
- Statistical mechanics (6 ec)
- Quantum mechanics (6 ec)

Courses offered on demand (sometimes as reading groups) are:
- Quantum theory of condensed matter (6 ec)
- Infinite dimensional Lie algebras (6 ec)
- Quantum probability theory (6 ec)
- Constructive quantum field theory (6 ec)

3.4 Mathematics and Education

This program combines mathematics that is especially useful for future mathematics teachers with the E-traineeships of the Instituut voor Leraar en School (ILS, Institute for Teacher and School). There are various routes to become a mathematics teacher. There is a direct route of one year mathematics and one year teacher training and a longer route that also involves a half year communication. In any case the idea is to combine mathematics and education as much as possible.

In the mathematics part there will be courses which are linked to school mathematics, have interesting historical contexts and which require an active participation of the students. Parts of the teacher training program will be combined with activities within the Mathematics department: the Dutch Kangaroo competition, the Ratio project for school curriculum development and the Wiskundig Denken (Mathematical thinking) project for pupils in the last years of secundary school.

Further information: Prof.dr. F.J. Keune.

4 Dutch Master Program in Mathematics

4.1 Program and schedule

In this chapter you find a list of all Master courses offered in 2008/2009 in the framework of the **DUTCH MASTER PROGRAM IN MATHEMATICS.** For descriptions of these courses and further details see: www.mastermath.nl .

You have to register for these courses at: www.mastermath.nl/registration .
Abbreviations:

- (LNMB)= these courses are organized by the Dutch Network on the Mathematics of Operations Research
- (MRI)= these courses are organized by the Mathematics Research Institute and are recommended only for advanced students who specialize in dynamics of differential equations.
- (3TU)= these courses are part of a joint MSc program in Applied Mathematics of the 3 Dutch technical universities (Technische Universiteit Eindhoven, Universiteit Twente and Technische Universiteit Delft)

4.2 Course Schedule Fall 2008

Monday	**Universiteit Utrecht**
10:15 - 12:00 hrs:	Discrete Optimization (6 cp)
Instructors:	Uetz, M. (Universiteit Twente)
10:15 - 14:45 hrs:	Introduction to Stochastic Processes (4 cp)
Instructors:	Adan, I.J.B.F. (Technische Universiteit Eindhoven) Boxma, O.J. (Technische Universiteit Eindhoven)
Venue:	This is a crash course. Meetings only on Mondays and Tuesdays September 1 September 2 September 8 September 9
10:15 - 12:00 hrs:	System and control (6 cp)
Instructors:	Polderman, J.W. (Universiteit Twente) Stoorvogel, A.A. (Universiteit Twente) Trentelman, H.L. (Rijksuniversiteit Groningen)
13:00 - 14:45 hrs:	Continuous Optimization (6 cp)
Instructors:	Still, G. (Universiteit Twente)
15:00 - 16:45 hrs:	Simulation (6 cp) (Instructors unknown)

Tuesday	**Vrije Universiteit**
10:15 - 13:00 hrs:	Algebraic Number Theory (8 cp)
Instructors:	Lenstra, H.W. (Universiteit Leiden) Stevenhagen, P. (Universiteit Leiden)
10:15 - 13:00 hrs:	Functional Analysis (8 cp)
Instructors:	Ran, A.C.M. (Vrije Universiteit) Jeu, M.F.E. de (Universiteit Leiden)
14:00 - 16:45 hrs:	Coding theory (8 cp)
Instructors:	Tilborg, H.C.A. van (Technische Universiteit Eindhoven) Willems, F. (Technische Universiteit Eindhoven)
14:00 - 16:45 hrs:	Partial Differential Equations (8 cp)
Instructors:	Vorst, R.C.A.M. van der (Vrije Universiteit)
Wednesday	**Universiteit van Amsterdam**
10:15 - 13:00 hrs:	Measure-theoretic probability (8 cp)
Instructors:	Spreij, P.J.C. (Universiteit van Amsterdam)
14:00 - 16:45 hrs:	Asymptotic statistics (8 cp)
Instructors:	Jongbloed, G. (Technische Universiteit Delft)
14:00 - 16:45 hrs:	Numerical Linear Algebra (8 cp)
Instructors:	Sleijpen, G.L.G. (Universiteit Utrecht) Gijzen, M.B. van (Technische Universiteit Delft)
Wednesday	**Universiteit Utrecht**
10:15 - 13:00 hrs:	Riemannian Geometry (8 cp)
Instructors:	Ban, E.P. van den (Universiteit Utrecht) Vandoren, S. (Universiteit Utrecht)
14:00 - 16:45 hrs:	Toric Geometry (8 cp)
Instructors:	Stienstra, Jan (Universiteit Utrecht)

4.3 Course Schedule Spring 2009

Monday	**Universiteit Utrecht**
10:15 - 12:00 hrs:	Advanced Linear Programming (6 cp)
Instructors:	Stougie, I. (Technische Universiteit Eindhoven)
10:15 - 12:00 hrs:	Advanced Modelling in Science (6 cp)
Instructors:	Peletier, M.A. (Technische Universiteit Eindhoven) Groesen, E.W.C. van (Universiteit Twente) Heemink, A.W. (Technische Universiteit Delft) Gils, S.A. van (Universiteit Twente) Molenaar, J. (Wageningen Universiteit)
10:15 - 12:00 hrs:	Applied Statistics (6 cp)
Instructors:	Bucchianico, A. di (Technische Universiteit Eindhoven) Lopuhaä, H.P. (Technische Universiteit Delft) Kallenberg, W.C.M. (Universiteit Twente) Alberts, W. (Universiteit Twente)
13:00 - 14:45 hrs:	Applied Finite Elements (6 cp)
Instructors:	Vegt, J.J.W. van der (Universiteit Twente) Segal, A. (Technische Universiteit Delft) Maubach, J.M.L. (Technische Universiteit Eindhoven)
13:00 - 14:45 hrs:	Nonlinear Systems Theory (6 cp)
Instructors:	Schaft, A.J. van der (Rijksuniversiteit Groningen) Scherpen, J.M.A. (Rijksuniversiteit Groningen)
13:00 - 14:45 hrs:	Scheduling (6 cp)
Instructors:	Hurink, J.L. (Universiteit Twente)
13:00 - 14:45 hrs:	Stochastic Differential Equations (6 cp)
Instructors:	Weide, J.A.M. van der (Technische Universiteit Delft) Wittich, Olaf (Technische Universiteit Eindhoven) Neerven, J.M.A.M. van (Technische Universiteit Delft)
15:00 - 16:45 hrs:	Queueing Theory (6 cp)
Instructors:	Scheinhardt, W.R.W. (Universiteit Twente)

Tuesday	**Universiteit van Amsterdam**
10:15 - 12:00 hrs:	Algebraic Geometry (8 cp)
Instructors:	Edixhoven, S.J. (Universiteit Leiden) Taelman, L. (Universiteit Leiden)
14:00 - 16:00 hrs:	Advanced graph theory (8 cp)
Instructors:	Schrijver, L. (CWI) Gerards, B. (CWI)
Tuesday	**Universiteit Utrecht**
10:15 - 13:00 hrs:	Dynamical System (8 cp)
Instructors:	Broer, H.W. (Rijksuniversiteit Groningen) Hanssmann, H. (Universiteit Utrecht)
14:00 - 16:45 hrs:	Variational and topologican methods for nonlinear partial differential equations (8 cp)
Instructors:	Hulshof, J. (Vrije Universiteit) Peletier, M.A. (Technische Universiteit Eindhoven) Berg, J.B van der (Vrije Universiteit)
Wednesday	**Universiteit van Amsterdam**
10:15 - 13:00 hrs:	Modular Forms (8 cp)
Instructors:	Geer, G.B.M. van der (Universiteit van Amsterdam)
14:00 - 16:45 hrs:	Algebraic Topology (8 cp)
Instructors:	Moerdijk, I. (Universiteit Utrecht) notbohm , D. (Vrije Universiteit)
Wednesday	**Universiteit Utrecht**
14:00 - 16:45 hrs:	Numerical Methods for PDEs (8 cp)
Instructors:	Stevenson, R. (Universiteit van Amsterdam)
Wednesday	**Vrije Universiteit**
10:15 - 13:00 hrs:	Stochastic Processes (cp)
Instructors:	Spieksma, F.M. (Universiteit Leiden)
14:00 - 16:45 hrs:	Time series (cp)
Instructors:	Vaart, A.W. van der (Vrije Universiteit)

5 Description of courses

Advanced Differential Geometry

Course ID: **WM065B** *6 ec* second semester prof. dr. J.H.M. Steenbrink

Website
www.ru.nl/wiskunde/wiskunde/wie_wat_waar/steenbrink_jozef

Teaching methods
- 2 hours tutorial with assignments

Prerequisites
Differential Geometry

Objectives
The student has a firm knowledge of the basic concepts of the theory of Lie groups and vector bundles. He knows the role they play in the study of the Dirac and Yang-Mills equations.

Contents
The course is intended to provide a working knowledge of those parts of the theory of Lie groups and vector bundles that are essential for a deeper understanding of both classical and modern physics

Subjects
- Lie Groups, Bundles, and Chern Forms
- Vector Bundles in Geometry and Physics
- Fiber Bundles, Gauss-Bonnet, and Topological Quantization
- Connections and Associated Bundles
- The Dirac Equation
- Yang-Mills Fields
- Betti Numbers and Covering Spaces
- Chern Forms and Homotopy Groups

Literature
Theodore Frankel: The Geometry of Physics. An Introduction. Cambridge University Press 1997. Chapter III.

Examination
Oral

Extra information
Part of the IMAPP Master in Mathematical Physics

Algebraic K-Theory

Course ID: **WM056B** *10 ec* prof. dr. F.J. Keune

Website
www.math.ru.nl/~keune

Teaching methods
- 28 hours lecture
- 28 hours tutorial

Objectives
The student has understanding of K0, K1 and K2 as functors of rings. He can relate these functors to more classical concepts as modules, determinants and reciprocity laws.

Contents
Finitely generated projective modules, especially over Dedekind domains. Stable equivalence of projective modules. The functor K0. The general linear group and the functor K1. The Steinberg group and the functor K2. Exact sequences in K-theory. Steinberg symbols on a field, in particular on a number field.

Literature
J. Milnor, *Introduction to Algebraic K-theory*, Princeton University Press, 1971.
B. A. Magurn, *An Algebraic Introduction to K-Theory*, Cambridge University Press, 2002

Examination
To be arranged with the participants.

Algebraic number theory

Course ID: **WM049B** *10 ec* prof. dr. F.J. Keune

Website
www.math.ru.nl/~keune

Teaching methods
- 28 hours lecture
- 28 hours tutorial

Objectives
The student has a level of understanding of algebraic number theory that is needed for the proof of the theorem of Kronecker and Weber on Abelian number fields. He/she can relate the theory to concrete number theory problems.

Contents
Algebraic number theory originates from problems about Diophantine equations and its main subject is the structure of algebraic number fields, i.e. finite extensions of the rationals. The main goal of the course is to obtain understanding of the theorem of Kronecker and Weber: every Abelian number field is contained in a cyclotomic field. For this some knowledge of Galois theory is required, especially in the second half of the course. There will be special sessions for students who still have to learn the Galois theory needed for this course. There will be special emphasis on the computation of important invariants of number fields: ideal class groups, groups of units.

Literature
Lecture notes; see lecturers website

Examination
Participants are expected to give talks on various sections and on solutions of exercises. From each participant a final presentation is required.

Algebraic topology

Course ID: **WM045A** *10 ec* dr. F.J.B.J. Clauwens

Website
www.math.ru.nl/~clauwens

Teaching methods
- 28 hours lecture
- 28 hours tutorial

Objectives
The student should be familiar with the notion of simplicial complex and be able to compute the homology of a polyhedron effectively in practical cases. He can list the properties of both simplicial and singular homology and can reproduce the derivation of these properties. He should also be able to explain important applications of these theories like the Lefschtez Fixed Point Theorem and the Generalized Jordan Curve Theorem.

Contents
The course starts with the geometry of 'simplicial complexes'. These are spaces built from simplces (= line segments, triangles, tetrahedra etc.). From the way these simplces fit together we construct certain abelian groups, the so called 'simplicial homology groups'. The definition enables one to compute these groups effectively.
Subsequently we study what happens when a space is dissected differently into simplices. This results in the theorem that homeomorphic spaces have isomorphic homology groups. Thus sometimes spaces can be shown to be essentially different by computing their homology. As a further application we show how a continuous map from a polyhedron to itself gives rise to a 'Leftschetz number' in the integers with the property that it vanishes for a map without fixed point.
Next we discuss the singular homology groups of an arbitrary topological space. These can not be computed directly from their definition. We will list a number of properties of these groups the so called 'Eilenberg-Steenrod axioms'. These properties characterize the theory on polyhedra. We will also see how these properties can be used to compute these groups.
Finally we prove a high-dimensional generalization of the Jordan Curve theorem: consider a continous map from the unit sphere in Euclidean space to the same Euclidean space; then the complement of its image has two open components with that image as a common border.

Literature
J.R. Munkres: `*Elements of Algebraic Topology'*, Addison-Wesley Publ. Comp., 1984.

Examination
Oral.

Axiomatic set theory

Course ID: **WM038A** *10 ec* Spring 2010 dr. W.H.M. Veldman

Teaching methods

- 28 hours lecture
- 28 hours tutorial

Objectives

The student comes to know the story of the formalization of set theory and some of its heroic results and open questions.

Contents

We explain how set theory started with *Cantor's diagonal* argument and his *continuum hypothesis*. We consider Zermelo's *Axiom of Choice* and some famous applications, including *Zorn's Lemma* and the *Banach-Tarski-paradox*. We then list the axioms given by Zermelo and Fraenkel and develop set theory from them including the theory of ordinals and cardinals. We go on to consider Gödel's *constructible sets* and his proof of the consistency of the continuum hypothesis. We try to obtain some idea of the *forcing method* developed by Cohen for proving the consistency of the negation of the continuum hypothesis. If time permits, we also discuss Mycielski's *Axiom of Determinacy*, and/or Aczel's *Anti-foundation axiom*.

Literature

K. Kunen, *Set Theory, an Introduction to the Independence Proofs*, North Holland Publ. Co., Amsterdam, 1980.
K. Devlin, *The Joy of Sets: an Introduction to Contemporary Set Theory*, Springer Verlag, New York, 1987
T. Jech, *Set Theory*, Springer verlag, New York, 1997
Y.N. Moschovakis, *Notes on Set Theory*, Springer Verlag, New York, 1994
R.L. Vaught, *Set theory: an Introduction*, Birkhäuser, Boston, 1985

Examination

After having completed and submitted a number of exercises, students have to pass an oral examination.

Coding Theory

Course ID: **WM005B** *6 ec* dr. W. Bosma

Website
www.math.ru.nl/~bosma

Teaching methods
- 28 hours lecture
- 28 hours tutorial

Contents
Coding theory deals with error correcting codes. These are constructed in order to reconstruct digital messages in which some bits may have been corrupted (think of noise added during the transmission of satellite photos or errors while reading from a CD). Error correction is achieved by adding redundant information, but this causes contradicting effects: adding bits slows down the transfer rate while it enhances the capacity of correcting errors.
In this course we will deal with algebraic aspects of linear (block-)codes which may be described as (sub-)vector spaces over a finite field. Wellknown and much applied constructions (quadratic residue-, BCH-, Reed-Muller- and cyclic codes) will be discussed as well as some recent constructions using algebraic geometry.
The most important algebraic methods are provided by the theory of finite fields and their polynomial rings.

Literature
Syllabus by dr. R.H. Jeurissen; ask lecturer

Examination
Oral

Cohomology and characteristic classes

Course ID: **WM034B** *6 ec* dr. F.J.B.J. Clauwens

Website
www.math.ru.nl/~clauwens

Teaching methods
- 28 hours lecture
- 28 hours tutorial

Prerequisites
Algebraic Topology. It is also useful to have some experience with differentiable manifolds and/or vector bundles.

Contents
Differentiable manifolds, vector bundles, Cohomology, Thom isomorphism, Stiefel-Whitney classes, Chern classes, Pontrjagin classes. Thom spaces, transversality, bordism theory.

Literature
John W. Milnor and James Stasheff: *'Characteristic Classes' Annals of mathematics studies 76,* Princeton University Press 1974 ISBN 0-691-08122-0

Examination
Yet to be determined

Commutative algebra

Course ID: **WM026A** *10 ec* second semester dr. A.R.P. van den Essen

Website
www.math.ru.nl/~essen

Teaching methods
- 28 hours lecture
- 28 hours tutorial

Objectives
The student is acquainted with the theory of modules over commutative rings. In particular he is familiar with the theory of Noetherian and Artin modules, tensor products, localization and all basic notions of commutative algebra.

Contents
What is commutative algebra about? To make this clear let's start with a k-vector space V, where k is a field. So V is a set equipped with an addition, which makes V into an abelian group, and a scalar multiplication with scalars from k. Furthermore the classical distibutive laws hold. If we replace k by an arbitrary commutative ring R we get a so-called R-module. This notion generalises most of the notions one meets during a Bachelor's study Mathematics. For example it will turn out that a Z-module is the same as an abelian group, a $k[x]$-module is the same as a k-vector space together with a linear transformation and an ideal I in a ring R is an example of a so-called R-submodule of R. Also the quotient ring R/I is an R-module etc. The theory of R-modules is much more complicated than the theory of vector spaces; many problems are still unsolved. The general philosophy is that the 'nicer' the ring R is, the more we know about its R-modules. The language of modules is an indispensable tool in nowadays Mathematics. In this course we discuss the most fundamental concepts and results of modern commutative algebra. Many of the notions introduced in this course will also be used in various other courses.
If you are planning to specialize in algebraic geometry, algebraic topology, number theory, computer algebra or polynomial mappings, this course is a must.

Literature
We follow the excellent book *'Introduction to commutative algebra'* by M.F. Atiyah and I.G. MacDonald.

Examination
The student has to make a series of exercises.

Coxeter Groups

Course ID: **WM051B** *6 ec* second semester prof. dr. G.J. Heckman

Website
www.math.ru.nl/~heckman

Teaching methods
- 28 hours lecture
- 28 hours tutorial

The course will be given every two years in the fall semester.

Prerequisites
Linear algebra, Symmetry.

Objectives
The student acquires a basic knowledge on group presentations and its geometric meaning, with a good deal of emphasis on the beautiful example of Coxeter groups.

Contents
Coxeter groups are groups with particular presentations coming from the geometry of polyhedral tessellations. This gives the subject both an algebraic and a (combinatorial and differential) geometric flavour. Coxeter groups show up naturally in various parts of mathematics, such as Lie theory (the Killing-Cartan classification of simple Lie groups), algebraic geometry (the period map of Enriques or K3 surfaces). Coxeter groups are also the mathematical frame work behind some of the geometric art of M.C.Escher.
In the course we will start with a basic introduction to Coxeter groups leading to the Tits theorem. Subsequently we will discuss Coxeter's paper on Escher's Circle Limit III and McMullen's paper on the link between Coxeter groups and the Hilbert metric.
The course is well suited for students from 'education' (especially the paper of Coxeter), 'symbolic computing' and 'mathematical physics' (for people interested in the simple Lie groups).

Literature
J.E. Humphreys, *Reflection groups and Coxeter groups*, Cambridge Studies Adv Math 29, CUP, 1990.
H.S.M. Coxeter, *The trigonometry of Escher's woodcut Circle Limit III*, Math Intellegencer, 1996.
C. McMullen Coxeter groups, *Salem numbers and the Hilbert metric*, preprint 2002, available from his webpage: www.math.harvard.edu ... etc

Examination
Oral exam / written assignment.

Crystallographic Groups

Course ID: **WB039B** *6 ec* second semester dr. B.D. Souvignier

Website
www.math.ru.nl/~souvi/onderwijs.html

Teaching methods
- 30 hours lecture
- 30 hours tutorial

Prerequisites
Linear algebra, Symmetrie

Objectives
- The student is acquainted with the mathematical background of crystallographic groups. Knowledge of the abstract notions of group theory is deepend and illustrated by explicit examples.
- The student knows how to analyze the symmetry properties of a given ornament and can generate periodic ornaments having a given symmetry group.
- He/she is familiar with lattices and point groups and knows how to combine these to crystallographic groups.
- He/she knows the basis of the theory of representations of crystallographic groups.

Contents
Crystallographic groups are the symmetry groups of periodic discrete point sets in Euclidean space, e.g. of (idealized) crystals. They have applications in various fields, e.g. in solid state physics, but also give rise to esthetically appealing decorations.
In this course we will look at the mathematical description of crystallographic groups which may serve as concrete illustrations of various abstract concepts in group theory. For example, the translations in a crystallographic group form a (vector) lattice. The factor group by this lattice is isomorphic with a finite subgroup of the orthogonal group and is called a point group. We will investigate how a crystallographic group is built up from these two ingredients, a lattice and a point group, and what the different possibilities are. This will require dealing with various important issues, such as presentations of groups by generators and relations or the computation of orbits and stabilizers.
With the acquired knowledge we will be able to construct the 2- and 3-dimensional crystallographic groups.
However, the concepts are dimension independent and we can look at higher dimensions. For example, 4- and 6-dimensional groups play an important role in the description of *quasicrystals* and the famous *Penrose patterns* can be constructed from a 5-dimensional 'cubic' lattice.
In the applications of crystallographic groups, their *representations* play an important role. We will see how these representations can be described and which concepts play a role herein.

Literature
Lecture notes on lecturers website

Examination
Assignments have to be handed in, and its topics have to presented in class. The course will be finishes with an oral exam.

Differential topology

Course ID: **WM035B** *6 ec* dr. M.H.A.H. Muger

Website
www.math.ru.nl/~mueger

Teaching methods
- 28 hours lecture
- 28 hours tutorial

Contents
Differential topology is a branch of general topology focussing on a class of particularly nice spaces, that is smooth manifolds. The study of the latter on one hand leads to exciting new problems (e.g. exotic smooth structures), but on the other hand a solid understanding of smooth manifolds is also fundamental for many further developments, like the study of manifolds with riemannian, symplectic or Poisson structures as well as for analysis on manifolds and index theory. Besides the fundamentals we will also discuss some connections with algebraic topology (degree, Euler number). For a more detailed overview see the table of content of the syllabus (mainly chapter 2).

Literature
Syllabus provided by the lecturer.
Supplementary references:
- V. Guillemin and A. Pollack: 'Differential topology', Prentice-Hall Inc.
- Bröcker and Jänich: 'Introduction to differential topology', Cambridge University Press, 1982 (or the original version in German)
- M. Hirsch: ′Differential topology′, Springer Graduate Texts in Math 33.

Examination
Oral

Elementary Number Theory

Course ID: **WM055B** *8 ec* prof. dr. F.J. Keune

Website
www.math.ru.nl/~keune

Teaching methods
- 40 hrs lecture

Prerequisites
Knowledge of the rudiments of groups and rings, as they were discussed in the first weeks algebra courses in bachelor Math educations.

Objectives
The student has knowledge of number theory, in particular reciprocity laws, with enable him to understand certain modern developments from an elementary point of view.

Contents
This course is primarily intended for a Master programme Mathematics and and Education, in particular the education variant of a Master course Mathematics. It consists of two parts. The first part consists of lectures including exercises. Reciprocity laws are the main theme; the quadratic, cubic and biquadrate reciprocity laws.
A number of subjects will be raised: unequivocal disintegration with applications, the structure of remaining class figures of the integers, arithmetical functions, finite numbers, Gauss and Jacobin sums. The goal of the course is to offer an insight in modern developments in the number theory form from an elementary perspective. Students will be working in small groups on a number theoretical subject, presenting their results in the second half of the course and writing an introductory about it.

The course is based on the first nine chapters of the book of Ireland and Rosen.

Literature
Kenneth Ireland, Michael Rosen, *A Classical Introduction to Modern Number Theory, Graduate Texts in Mathematics*, Vol. 84, Springer-Verlag, ISBN 0-387-97329-X

Examination
Final assessment is based half on homework and half on a lecture about a subject of choice.

Fundamental questions

Course ID: **WB028B** *6 ec* dr. W.H.M. Veldman

Teaching methods

- Lectures

Objectives

The students will learn that questioning the nature of mathematics and the exact foundations of mathematical arguments are interesting in itself and highly important to the didactics of mathematics.

Contents

Possible subjects are the parallell postulate, ratio and real numbers, Cantor's diagonal argument, Cantor-Shroeder-Bernstein's thesis, the continuum hypothesis, the choice axiom, Banach and Tarski's paradox and infinite plays.
There will be some attention for the intuitionistic approach on these subjects.

Literature

Yet to be determined

Examination

Yet to be determined

Extra information

This course is meant for students in the E and EC variant and other students who want to orientate in the foundations of mathematics. It will be given on request.

Galois Theory

Course ID: **WM025B** *6 ec* prof. dr. F.J. Keune

Website
www.math.ru.nl/~keune

Teaching methods
- 28 hours lecture
- 28 hours tutorial

Objectives
The student understands the role of group theory for the solution os algebraic equations in one unknown. He is able to formulate problems about algebraic equations in terms of field extensions.

Contents
Automorphisms of field extensions. Normal and separable extensions. The main theorem of Galois theory. The Galois theory of cyclotomic extensions. Solubility of equations and groups. Solubility of special types of equations.

Literature
view lecturers website

Examination
Written exam

Graph theory

Course ID: **WM006A** *10 ec* dr. W. Bosma

Website
www.math.ru.nl/~bosma

Teaching methods
- 28 hours lecture
- 28 hours tutorial

Contents
In its simplest form a graph is a set (of points/vertices) with a symmetric, non-reflexive relation, indicating whether two points are connected (by an edge) or not. Graphs are used e.g. to model networks for transport or communication as well as for assignment problems. In this course we will look at the fundamental concepts of graph theory, paying attention to combinatorial, algebraic, topological, but also to algorithmic aspects. Starting from elementary notions like distances, cycles, trees, connectedness or regularity we will proceed to coloured and labeled graphs which enlarge the field of possible applications. For example, in a transportation network the edges of a graph will be labeled by the costs arising on that connection.
Many natural questions on graphs give rise to interesting algorithmic methods, for example to find the distance (i.e. the shortest/cheapest path) between two points, to find the diameter of a graph (the largest distance between two points) or to determine short or long cycles in a graph. Furthermore, the connection of graph theory with other areas of algebra will be stressed, since many important graphs arise from groups, codes or designs.

Literature
Check lecturers website

Examination
to be consulted with lecturer

Group Theory

Course ID: **NM028B** *9 ec* second semester prof. dr. G.J. Heckman

Teaching methods

- 56 hrs lecture
- 28 hrs problem session

Prerequisites

Bachelor course 'Introduction to Group Theory' is a must

Objectives

- The student acquires basic knowledge with the role of symmetry in quantum mechanics (e.g. Kramers degeneration and selection rules)
- The student is familiar with Wigner's Theorem on isomorphisms of ray space
- The student has familiarity with the concept of induced representations
- The student understands the unitarity spectrum of crystallographic groups and the Poincare group
- The student is familiar with the concept of universal enveloping algebra of a Lie algebra
- The student understands the classification of irreps of SU(2) and SU(3)
- The student has a clear understanding of the role of SU(2) and SU(3) for physics

Contents

Group Theory plays a pivotal role in problems of mathematics and physics where one encounters symmetry of some sort. For example, molecular symmetry (if sufficiently rich) yields collision of spectral lines (spectral degeneration) and selection rules. Another example is the prominent role of the Lie group SO(3) in quantum mechanics, in particular in the exact treatment of the H-atom. These two applications have already been discussed in introductory courses in the bachelor study (Inleiding Groepentheorie and Inleiding Liegroepen).
The present course consists of three parts. The first topic is of a general nature about the use of groups in quantum mechanics (Wigner's theorem, Kramers degeneration, central extensions, Wigner-Eckhart theorem and selection rules).
The second part concerns induced representations with emphasis on the case of crystallographic groups (for solid state physics) and the Poincare group (for elementary particle physics).
Part three concerns Lie algebraic methods with special interest for the eightfold way Lie algebra SU (3) whose representation theory underlies the quark formalism in elementary particle physics.

Literature

- Will be handed out during the course

Examination

Oral exam

Extra information

This course is given biennually

Groups and representations

Course ID: **WM010A** *10 ec* dr. B.D. Souvignier

Website
www.math.ru.nl/~souvi

Teaching methods
- 28 hours lecture
- 28 hours tutorial

Prerequisites
Linear algebra, groups, rings and fields (Lineaire Algebra, Symmetrie, Ringen en Lichamen)

Objectives
The student is acquainted with the basic theory of group representations and is able to deal with representations and characters in concrete examples. He knows how interesting properties of groups can be derived from their representations and characters and how the information required can be computed explicitly.

Contents
In order to compute in an abstract group we require an explicit realization of the group elements. One possibility for such a realization is to view the group as a group of matrices defined as the image of a homomorphism from the group to a matrix ring. Such a homomorphism is called a representation of the group. The analysis of groups via their representations is a powerful tool which contributes to many and varied problems, such as the classification of finite simple groups, the theory of Lie groups or the determination of possible tilings of planes or spaces. In this course we will both deal with the theory of group representations and with algorithmic methods that allow to apply the theory to explicit examples. In particular we will see how to construct representations, how to decompose representations into irreducible constituents and how to utilize the distillation of representations to their characters.

Literature
- M.Burrow: *Representation Theory of Finite Groups*, Academic Press, 1965
- J.H.Conway, R.T.Curtis, S.P.Norton, R.A.Parker, R.A.Wilson: *Atlas of Finite Groups*, Clarendon Press, 1985
- C.W.Curtis, I.Reiner: *Methods of Representation Theory*, Wiley-Interscience, 1981
- I.M.Isaacs: *Character Theory of Finite Groups*, Academic Press, 1976
- G.James, M.Liebeck: *Representations and Characters of Groups*, Cambridge UP., 1993
- A syllabus will be provided.

Examination
The students will be asked to hand in exercises and to present their solutions. The course will be rounded off by an oral examination.

Hilbert spaces and quantum mechanics

Course ID: **WM053B** *6 ec* first semester prof. dr. N.P. Landsman

Website
www.math.ru.nl/~landsman/HSQM.pdf

Teaching methods
- 28 hours lecture
- 28 hours tutorial

Prerequisites
Mathematics students: Analysis 1, 2 & 3
Physics students: Analysis 1: Quantum mechanics 1 & 2

Objectives
The student is able to understand the abstract theory of Hilbert spaces and linear operators (bounded as well as unbounded) and can apply this theory to quantum mechanics.

Contents
On one hand, this course gives an introduction to the mathematical theory of Hilbert spaces, which lie at the basis of modern analysis. On the other hand, quantum mechanics will be discussed in a mathematically rigorous way as a key application of the theory of Hilbert spaces.

Subjects
- Historical introduction
- Metric spaces, normed spaces, and Hilbert spaces
- Linear operators and functionals
- Compact operators
- Closed unbounded operators
- Spectral theory for closed unbounded operators
- Quantum mechanics and Hilbert space
- Spectral theory for selfadjoint operators
- Quantum logic

Literature
Lecture notes (preliminary version available from website)

Examination
Oral

History and Foundations of Quatum Mechanics

Course ID: **NB063B** *6 ec* first semester drs. M.P. Seevinck
prof. dr. N.P. Landsman

Teaching methods
- 32 hrs lecture
- 32 hrs problem session

Examination
- Wekelijkse schrijfopdrachten
- Presentatie
- Schriftelijk tentamen en/of het schrijven van een academisch artikel

Hypergeometric functions and Representation theory

Course ID: **WM061B** *6 ec* first semester prof. dr. H.T. Koelink
prof. dr. G.J. Heckman

Website
www.math.ru.nl/~koelink/ http://www.math.ru.nl/~heckman

Teaching methods
- 28 hours lecture
- 28 hours tutorial

Prerequisites
Symmetrie, Differentiaalvergelijkingen, Topologie

Objectives
The student is familiar with the complex analytic aspects of differential equation and with the hypergeometric equation. The student is familiar with some group representations, Gelfand pairs and spherical functions. Finally, the student understands the relation between these two subjects.

Contents
The first half of the course, Hypergeometric Functions, entitled deals with ordinary linear differential equations in a complex domain. The existence and uniqueness theorem in the holomorphic setting is discussed. The notions of regular, singular and regular singular points is treated. The monodromy group of such an equation will be explained. The example of the hypergeometric equation will be taken along throughout the exposition to illustrate the various concepts. We will end the first half with a
discussion of the Riemann-Hilbert problem.
The second half of the course, Representation Theory, deals with harmonic analysis on Gelfand pairs. A Gelfand pair is a pair (G,K) with a group G and a compact subgroup K with suitable properties. Spherical functions are functions on such a group G that can be characterised in several equivalent ways. For specific examples the spherical functions can be expressed in terms of hypergeometric functions entailing several properties for these functions in a representation theoretic way.

Literature
lecture notes Hypergeometric Functions by G.J. Heckman and Analyse Harmonique sur les Paires de Guelfand et les Espaces Hyperboliques by J. Faraut.
(available through blackboard)

Examination
Oral

Introduction to Differential Geometry

Course ID: **WM060B** *6 ec* first semester dr. F.J.B.J. Clauwens

Website
www.math.ru.nl/~clauwens

Teaching methods
- 28 hours lecture,
- 28 hours tutorial

Prerequisites
Some familiarity with the topological way of thinking is highly recommended.
A good knowledge of Calculus and (multi)linear algebra is indispensable.

Objectives
The student should should be able to explain the concepts listed below and should be able to use them to formulate parts of classical mechanics and of General Relativity in differential geometric terms.
- Topological space, chart, atlas, differentiable manifold, smooth map
- Tangent vector, tangent space, derivation, vector field, Lie derivative
- Cotangent vector, cotangent space, covector field
- Tensors and tensor fields
- Differential form, exterior differentiation
- Partition of Unity, volume form, integration on manifolds, Stokes theorem
- (Semi)-Riemann structure, affine connection, Christoffel symbols, Levi-Civita connection
- Curvature tensor, sectional curvature, Bianchi identities
- Parallel displacement, geodesics, exponential map, normal coordinates

Contents
A topological space is just a set together with some extra structure which enables one to talk about continuity of functions.Likewise a differentiable manifold is a set together with some extra structure which enables one to talk about differentiable functions and their derivatives. The idea of directional derivative leads to a construction which attaches a vector space to each point of the manifold, the so called tangent space. Using techniques of Linear Algebra one constructs numerous other vector spaces from this tangent space. Concepts like gradient, curl, divergence and their relations can thus be understood in a very general context. This provides a geometrical language in which one can describe configuration spaces and phase spaces, and in which one can formulate Maxwells theory of electromagnetism ans Einsteins theory of general relativity.

Literature
- Theodore Frankel: The geometry of physics, Cambridge Univeristy Press, ISBN 10-0-521-53927-7
- There wil also be lecture notes available in pdf form (some 100 pages) with material complementary to the above book.

Examination
Oral exam

Introduction to Partial Differential Equations

Course ID: **WB046B** *6 ec* second semester prof. dr. H.T. Koelink

Website
fa.its.tudelft.nl/~sweers

Teaching methods
- Lectures
- Tutorials

Contents
A partial differential equation describes a relation between the partial derivatives of an unknown function and given data. Such equations appear in all areas of physicsan engineering. More recently the use of PDEs in models in biology, pharmacy, imageprocessing, finance etc. have increased strongly. Since the origin of these models is very diverse and the results should be applicationdriven, the analysis of PDEs has many facets. The classical approach focused on finding explicit solutions. Since numerical methods and fast computers became available, the modern approach is more oriented to the application of functional analytic methods in order to find existence and uniqueness results and to show that solutions depend continuouslyon the given data. Having existence, uniqueness and stability under perturbations, a numerical method may be implemented to find an approximation of the solution one is interested in. The present course will be an introduction to the field. The elementary classical results will be explained and we will touch some of the more modern aspects.

Subjects
- Introduction: some elementary models will be explained and different types of PDEs will be classified.
- First order equations: the method of characteristics, conservation laws and shock waves.
- Linear second order equations: the heat equation, the laplace equation and the wave equation are classical second order models.
- The wave equation for one space dimension: The Cauchy problem and d'Alembert's solution.
- Seperation of variables. For special domains and special PDEs one may split the problem into a set of ODEs.
- Sturm-Liouville equations. Parameter dependent boundary value problems for ODEs.
- Elliptic equations. The maximum principle and uniqueness.
- Integral representations. For some special cases Green functions give an almost explicit solution.
- Equations in higher dimensions: the classification in parabolic, elliptic and hyperbolic equations. Some explicit solutions.
- Variational methods. Introducing the weak formulation.
- Some numerical methods: a first glance at finite differences and finite elements.

Literature
The course is based on the book *An Introduction to Partial Differential Equations* by Pinchover and Rubinstein published by Cambridge University Press.

Examination
Written exam

Intuitionistic mathematics

Course ID: **WM037A** *10 ec* second semester dr. W.H.M. Veldman

Teaching methods
- 28 hours lecture
- 28 hours tutorial

Objectives
De student learns that mathematics may be developed in other ways than the usual one, in particular, along the lines indicated by the famous Dutch mathematician L.E.J. Brouwer.

Contents
In this course we consider the criticism L.E.J. Brouwer (1881-1966) exercised on many results of classical real analysis, and explain why he refused to use the *principle of the excluded middle* in his own mathematical proofs. Brouwer not only wanted to restrict the logic of mathematical arguments but also proposed some new axioms. We will see that his new mathematics contains many delightful and convincing results.
We also treat *intuitionistic logic* as formalized by Heyting and Gentzen.
We will compare Brouwer's point of view with other conceptions of constructive mathematics.

Literature
- A. Heyting, *Intuitionism, an Introduction*, North Holland Publ. Co., Amsterdam 1971
- E. Bishop, D. Bridges, *Constructive Analysis*, Springer Verlag, New York etc.,1985
- D. Bridges, F. Richman, *Varieties of Constructive Mathematics*, Cambridge UP, 1987
- A.S. Troelstra. D. van Dalen, *Constructivism in Mathematics*, Volumes I and II, North Holland Publ. Co., 1988

Examination
After having completed and submitted a number of exercises, students have to pass an oral examination.

Lattice Theory

Course ID: **WB050C** *6 ec* first semester prof. dr. M Gehrke

Website
www.math.ru.nl/~mgehrke

Teaching methods
- 28 hours tutorial
- 28 hours lecture

Prerequisites
Ringen en lichamen 1

Objectives
After this course the student is familiar with the notions of ordered algebraic structures, including lattices and Boolean algebras, and has seen the connection to various topics in algebra, analysis, and computer and information science. He or she is familiar with Formal Concept Analysis and Birkhoff Duality for finite distributive lattices, and has seen several applications including one to knowledge representation.

Contents
We will introduce lattices both as partially ordered sets and as algebras. From the partial order point of view, we treat Hasse diagrammes, complete lattices, Galois connections and Formal Concept Analysis. From the algebraic point of view, we treat the homomorphism theorems, special classes such as modular, distributive, and Boolean algebras as well as the representation theory for finite lattices and its relation to classical propositional logic.

Literature
B.A. Davey & H.A. Priestley, *Introduction to Lattices and Order*, 2nd edn (CUP 2002).

Examination
oral

Lattices and crystallographic groups

Course ID: **WM009A** *10 ec* dr. B.D. Souvignier

Website
www.math.ru.nl/~souvi

Teaching methods
- 28 hours lecture
- 28 hours tutorial

Prerequisites
Linear algebra, groups, rings and fields.

Contents
Crystallographic groups are the symmetry groups of lattices, i.e. of periodic discrete point sets in Euclidean space.They have applications in various fields, e.g. in solid state physics, but also give rise to esthetically appealing decorations.
In this course we will look at the basic notions of crystallographic groups from an algorithmic point of view. Concepts like lattices, point groups, group extensions and representations of groups will always be accompanied with algorithms to deal with them and with objects to which the algorithms can be applied - thus giving a hands-on experience.
As a consequence we will be able to determine the essentially different possibilities to decorate a frieze or a wall and the different forms of 3-dimensional crystals. The concepts which are introduced can be generalized to arbitrary dimensions and provide the basis to analyze more complicated structures like quasicrystals or bio-macromolecules.

Literature
A syllabus will be provided by the lecturer

Examination
written exam

Lie algebras

Course ID: **WM062B** *6 ec* first semester prof. dr. G.J. Heckman

Website
www.math.ru.nl/~heckman

Teaching methods
- 42 hours lecture
- 14 hours tutorial

Prerequisites
Symmetry or Introduction Group Theory

Objectives
The student becomes familiar with the following subjects:
- Poisson algebras
- Universal enveloping algebras
- The representation theory of SL(2)
- Representations via constructions of linear algebra
- Reductive Lie algebras.
- Verma representations
- The representation theory of SL(3)
- Physical applications: Spin and quarks
- The Weyl character formula Spherical harmonics, and SO(n)
- Physical application: The Kepler problem.

Contents
In this course we discuss the mathematics of Lie algebras and their representations. The basic examples are the Heisenberg algebra, the special linear algebra SL(n) and the orthogonal algebra SO(n). For each of these algebras we discuss the physical relevance which lie mainly in the realm of particle physics. We also discuss the link with invariant theory, an important subject in geometry. The course is interesting for students in both physics and mathematics, and standard for students in mathematical physics. The material of the course is useful for the courses "Beyond the Standard Model" and "Introduction to String Theory" of Prof. dr. B. Schellekens.

Literature
is given in the class

Examination
Oral exam

Linear Programming

Course ID: **WB044C** *3 ec* first semester

Website
www.ru.nl/wiskunde/wiskunde/wie_wat_waar

Teaching methods
- 28 hours lecture
- 28 hours tutorial

Prerequisites
linear algebra 1,2,3,4

Objectives
The student is able to model a practical problem to a linear programming problem, and knows several methods to solve this problem.

Contents
Starting with examples, the students will be made familiar with linear programming, which is concerned with maximalizing or minimalizing a linear function in multiple variables, in an area marked off by linear inequalities. Some time will be spend on modelling practical problems. An example: A fruit company imports boxes of pineapples and boxes of bananas by boat. A box of pineapples weights 9 kg and is 1 m3 big. A box of bananas only weights 6 kg but is twice as big as a box of pineapples. There is room on the boat for 800 m3 and 3600 kg. Once arrived the company makes a profit of 50 euro on a box of pineapples and a profit of 40 euro on a box of bananas. How many boxes of each should the company take on one trip to maximize the profit? The model: Let p be the number of boxes of pineapples and b the number of boxes of bananas, they take on their boat. Of course they can only take a positive amount of boxes with them, so we may assume that p >= 0 and b >= 0. Secondly, the volume of the boxes should not exceed 800 m3, so p+2b <= 800. Furthermore the weight of the boxes should not exceed 3600 kg, so 9p+6b <= 3600. These are the linear constraints of our problem. Finally they want to maximize the profit so they have to maximize the function 50p+40b. There are a number of methods to solve this kind of problems, namely: the simplex method, the ellipsoid method, some interior point methods. These will be discussed during the course. After following this course it should be no problem to get the following answer to the question of the fruit company: they should take 200 boxes of pineapples and 300 boxes of bananas.

Literature
Matouvek en B. Gartner, Understanding and using Linear Programming, Springer-Verlag, 2007, ISBN 978-3-540-30697-9.

Examination
The examination consists of three parts, namely:
1. Homework to be handed in weekly, 15% of the final grade.
2. A final project, consisting of a practical problem that needs to be modelled and solved using linear programming, 15% of the final grade.
3. A written exam, 70% of the final grade.

Machine Learning

Course ID: **NM048B** *6 ec* first semester prof. dr. H.J. Kappen
dr. W.A.J.J. Wiegerinck

Website
www.snn.ru.nl/~wimw/collegeML.html

Teaching methods
- 40 hrs lecture

Prerequisites
- Voortgezette Kansrekening , Markov ketens, Toegepaste wiskunde 1,Machine Learning and Neural Networks.

Contents
This course is an advanced course on machine learning and neural networks from a probabilistic point of view. The course is a continuation of the course Introduction to Machine Learning and Neural Networks.
Some applications in vision and robot control.
The course is intended for master students in physics and mathematics.

Subjects
- Short recapitulation (MK Ch. 2, 3) of
- Bayesian posterior estimate for Gaussian distribution (MK Ch. 24)
- Graphical models (MK Ch. 21.1, 16, 26, Bishop Ch 8)
- Examples of useful probability distributions (MK Ch. 23)
- Laplace method (MK Ch. 27)
- Variational methods (MK Ch. 33)
- Model comparison (MK Ch. 28 + ev. illustratie MLPs MK Ch. 44)
- Monte Carlo methods (MK Ch. 29, 30)
- Binary netwerken, Markov processen, ergodiciteit (diktaat)
- Ising model (MK Ch. 31)
- Boltzmann Machines, mean field theorie (MK Ch. 43, diktaat)
- Independent component analysis (MK Ch. 34)
- Gaussian processes (MK Ch. 45)

Literature
- David MacKay, *Information Theory, Inference and Learning Algorithms*, Cambridge University press. The entire book can be viewed on-screen at www.inference.phy.cam.ac.uk/mackay/itila/book.html
- several handouts will be distributed during the course

Examination
Written exam

Model Theory

Course ID: **WM036B** *6 ec* first semester dr. W.H.M. Veldman

Teaching methods
Lectures

Objectives
The student becomes familiar with some results and techniques from model theory, the most important meeting point between mathematics and mathematical logic.

Contents
In mathematics one often studies the class of structures satisfying a given set of formal axioms, for instance the class of groups, the class of fields, or the class of linear orders. In *Model theory* one starts to restrict oneself to the still rather general case that the axioms are formulated in a *first-order* or *elementary* language. This means that, when interpreting the formulas of such a language, one only quantifies over the domain of a given structure, and not, for instance, over the power set of the domain. One then asks questions like: given a structure, is it possible to axiomatize it, that is, is it possible to indicate a *not too difficult* set of formulas valid in the structure such that every formula valid in the structure logically follows from the formulas in the set. Or: given structures A, B, under what circumstances are A, B *elementarily equivalent*, that is, when do they satisfy the same elementary formulas? Or: given a set of formulas, how many countable structures do there exist satisfying all formulas in the set?
Model theory at its best is a delightful blend of abstract and concrete reasoning.

Literature
- C.C. Chang, H.J. Keisler, *Model Theory*, North Holland Publ. Co., Amsterdam, 1977
- G.E. Sacks, *Saturated Model Theory*, Benjamin, Reading, Mass.,1972
- B. Poizat, *Cours de théorie des Modèles*, Nur al-Matiq wal-Marífah, 1985
- W. Hodges, *A shorter Model Theory*, Cambridge University Press, 1997

Examination
After having completed and submitted a number of exercises, students have to pass an oral examination.

Extra information
Intended for bachelor students who do not want to do the whole 10 ec course.

Modular Forms

Course ID: **WM030C** *6 ec* first semester dr. B.R. Kane

Website
www.math.ru.nl/~bkane

Teaching methods
- Lectures

Contents
Modular forms are analytic functions on the complex upper half plane which have a very specific transformation behaviour under substitutions which are given by fractional linear transformations with integral coefficients. Modular forms are closely connected with elliptic curves. They are used to construct modular curves, i.e. spaces parametrizing isomorphism classes of elliptic curves with some extra structure, such as a rational point of given order. Wiles has recently shown that every 'stable'elliptic curve over the filed of rational numbers can be paramatrized by modular forms. This is a crucial step in his proof of Fermat's Last Theorem. The course will give an elementary introduction to modular forms; it presupposes some familiarity with elliptic curves and with complex function theory of one variable. It will be useful if some knowledge about Riemann surfaces is present. A text, based on an MRI Master Class course given in 2000-2001 can be found on my website. This course has a load of 8 EC. It can be chosen at any time. Typically, students study the subject themselves, and meet me once a week for about half an hour to ask questions.

Literature
- R.C. Gunning, *Lectures on modular forms*. Annals of Mathematics Studies 48, Princeton Univ. Press 1962
- A. Knapp, *Elliptic curves*. Mathematical Notes 40, Princeton Univ. Press 1992
- S. Lang, *Introduction to modular forms*. Springer 1976
- J.-P. Serre, *Cours d'arithmetique,* Presses Universitaires de France 1970
- G. Shimura, *Introduction to the arithmetic theory of Automorphic Functions*. Princeton Univ. Press 1971

Examination
Yet to be determined

Operator algebras

Course ID: **WM063B** *6 ec* second semester dr. M.H.A.H. Muger

Website
www.math.ru.nl/~mueger/OA2009.html

Teaching methods
- 30 hrs lectures
- 30 hrs exercise classes

Prerequisites
Analyse I+II and at least one of the following courses: "Hilbert spaces and Quantum Mechanics" (Landsman), "Functional analysis" (Ran and de Jeu, landelijke master). In particular, we expect some familiarity with bounded, compact and unbounded operators on Hilbert spaces, incuding the spectral theorem.

Objectives
The students are familiar with the basics of C*-algebras and von Neumann algebras, allowing them to specialize further or to apply operator algebras in the context of infinite quantum systems.

Contents
After a brief recollection of Hilbert space and operator theory, we will consider the basic facts of C*-algebra theory, including some examples. We will then turn to von Neumann algebras (certain special C*-algebras) and their classification in terms of projections and modular theory. Time permitting, we may briefly touch upon some other subjects, cf. below, taking into account the interests of the participants.

Subjects
- Reminder on Hilbert spaces and operator theory
- Banach algebras, spectrum
- commutative C*-algebras
- ideals, quotients, homomorphisms
- states and representations
- weak topologies, density theorems
- von Neumann algebras
- projections, type classification some modular theory, with application to classification

Possible further subjects, to be decided:
- some K-theory of C*-algebras,
- classification of AF-algebras some subfactor theory
- applications to quantum spin systems and quantum field theory

Literature

- Excellent survey of the whole field, but not many proofs: Bruce Blackadar: Operator algebras. Springer, 2006. (Encyclopedia of Mathematical Sciences vol. 122.)
- More specialized, but much deeper coverage: Masamichi Takesaki: Theory of Operator Algebra I. Springer, 1997, 2002. (Encyclopedia of Mathematical Sciences vol. 124.
- Example-based approach to C*-algebras: Kenneth R. Davidson: C*-Algebras by example. American Mathematical Society, 1996.

Examination

Homework and written exam

Philosophy of mathematics

Course ID: **WM040B** *3 ec* on request dr. W.H.M. Veldman

Teaching methods
- 30 hrs lecture

Objectives
The student will learn to see that the question about the nature of mathematics is one of the most important questions in philosophy, and that meticulous mathematical thinking and philosophical contemplation can stimulate each other.

Contents
During the course we discuss: Plato's Ideas and the place of the mathematical objects, Aristotle's view, Kant's view on the nature of mathematical statements, Frege and Russell's logics, Russell's paradox, Cantor's discoveries, Brouwer's intuitionist criticism, Goedel's incompleteness thesis, Goedel's Platonism, Wittgenstein's thoughts.

Literature
The students will be given home texts that will be discussed in the lectures, during which one will be able to ask questions, or have discussions.

Examination
The student studies various texts of choice, and is assessed about them orally.

Polynomial mappings

Course ID: **WM027A** *10 ec* dr. A.R.P. van den Essen

Website
www.math.ru.nl/~essen

Teaching methods
- 28 hours lecture
- 28 hours tutorial

Prerequisites
Commutative Algebra

Objectives
The student is familiar with the theory of derivations and its applications to the study of polynomial automorphisms. He has an overview of the various main problems in the field.

Contents
A polynomial mapping from **C**n to itself is a mapping whose components are polynomials in *n* variables with complex coefficients. A linear mapping is an example, namely in this case each component is a linear polynomial in the variables. Linear mappings play a fundamental role when one wants to solve systems of linear equations. The following results were obtained in the linear algebra courses. A linear mapping is invertible if the determinant of the corresponding matrix is invertible in **C**. If a linear mapping from **C**n to itself is injective, then it is invertible and the inverse is again a linear mapping. Every invertible matrix is a product of elementary matrices. In this course we investigate how (if possible) these results can be generalised to polynomial mappings. The research on this kind of questions has led to interesting new results and relations with problems in other areas of mathematics(such as dynamical systems and cryptography) but also it gave rise to various famous conjectures and unsolved problems. These attractive problems, such as the Jacobian conjecture which is an attempt to generalise the first item above, have made the field of polynomial mappings into a rapidly growing research area. We discuss the basic results of the theory as well as several of the very recent new discoveries which take place at the forefront of research in this exciting part of Mathematics.

Literature
We study parts from the book *'Polynomial Automorphisms and the Jacobian Conjecture'*, by Arno van den Essen, Vol. 190 in Progress in Mathematics, Birkhauser (2000).

Examination
The student can make a series of exercises or do research in a topic and write a report on it.

Probability Theory

Course ID: **WB022B** *6 ec* second semester dr. H.W.M. Hendriks

Website
www.math.ru.nl/~hendriks

Teaching methods
- 28 hours lecture
- 28 hours tutorial

Prerequisites
The introductory courses Probability, Advanced Probability, Introduction in Statistics and the course Measure and Integral.

Objectives
The student has mastered the translation between measure and integral theoretical notions and probabilistic notions. Understanding and skill in the different notions of convergence, and the notion of independence of families of sigma-algebras. Understanding and skill in conditional expectation and familiarity with martingales.

Contents
The Kolmogorov axiomatics of probability theory. Expected value, moments, Jensen inequality. Types of convergence. Independence of sigma-algebras. Zero-One Laws, Laws of large numbers, Central limit theorem. Conditional expectation with respect to a sub-sigma-algebra. Martingales in discrete time and basic properties.

In this course we treat the foundation of modern probability theory as an application of measure and integration theory. The central theorems, like the lows of large numbers and the central limit theorem are treated in great generality.

Moreover the notions of conditional expectation and conditional probability will be broadened. We will pay attention to martingales, which are an important ingredient in the description of financial price processes.

Literature
Durrett, R., Probability: Theory and Examples, Duxbury

Examination
oral

Quantum Gauge Theory

Course ID: **WM064B** *6 ec* second semester dr. W.D. van Suijlekom

Website
www.math.ru.nl/~waltervs

Teaching methods
- 30 hours lectures

Prerequisites
Quantum Field Theory

Objectives
The student is acquainted with the mathematical structures underlying perturbative quantum gauge theories, such as the BRST-formalism, and can apply them to the renormalization of Yang-Mills gauge theories.

Contents
We introduce the mathematical formalism that describes perturbative quantum gauge theories.
We focus on Yang-Mills theories which are the building blocks of the extremely successful Standard Model of high-energy physics.
The symmetry of the Lagrangian on the classical level under gauge transformations imply so-called Ward identities between the Green's
functions on the (formal) quantum level. Mathematically, these identities are nicely captured by the BRST-formalism.
We describe renormalization of quantum gauge fields and encounter anomalies as a breaking of the BRST-symmetry.
If time permits, we finish by renormalizing gauge theories with (spontaneous) symmetry breaking.

Literature
S. Weinberg, *The quantum theory of fields. Vol. II Modern Applications,* Cambridge University Press (1996), ISBN 0521 55002

Examination
Oral

Extra information
This course will be given biennually

Stochastic calculus

Course ID: **WM019B** *6 ec* dr. J.D.M. Maassen

Website
www.math.ru.nl/~maassen

Teaching methods
- 28 hours lecture
- 28 hours tutorial

Prerequisites
Measure and integral, Probability theory

Objectives
The student is able to work out solutions of diffusion equations by Ito calculus, and to determinate their properties.

Contents
In the 1950's K. Itô developed a differential calculus to deal with random functions. This fine piece of functional analysis has found application in a wide range of fields, extending from electronic engineering via quantum field theory and population biology to the stock market. In this course Itô diffusions are constructed, their calculus is developed, and several applications are treated. The course will be taught if sufficiently many participants show up.

Subjects
- Brownian motion
- The Itô integral
- Stochastic differential equations
- Applications to potential theory
- Path integrals
- The Kalman filter
- The Black and Scholes option price

Literature
Lecture notes on lecturers website

Examination
Oral exam

The Structure of Spacetime

Course ID: **WM058B** *6 ec*

dr. W.D. van Suijlekom
dr. E.J. Hawkins

Website
www.math.ru.nl/~waltervs

Teaching methods
- 28 hours lectures
- 28 hours tutorial

Prerequisites
Tensoren en Toepassingen; Inleiding Algemene Relativiteitstheorie

Objectives
The student has a conceptual understanding of the mathematical structure of General Relativity and is able to understand the research literature on gravitational physics.

Contents
We introduce the mathematical techniques necessary for applying Einstein's general theory of relativity. These include the concepts of manifolds, curvature, symmetries, differential forms, and conformal/causal structure. Using these, we will cover singularity theorems, integral theorems, and applications to cosmology and the death of stars.

Literature
- R.M. Wald, *General Relativity,* University of Chicago Press, 1984, ISBN 9780226870335

Examination
Oral exam

Extra information
This course is given biennually. Next occasion will be 2009/2010

Theory of recursive functions: computability, unsolvability and unprovability

Course ID: **WM039A** *10 ec* 2009/2010 dr. W.H.M. Veldman

Teaching methods
- 28 hours lecture
- 28 hours tutorial

Objectives
The student learns the notion of a computable function, that is of central importance both for mathematics and for theoretical computer science.

Contents
We describe various approaches of the class of partial computable functions from *N* to *N*, due to Turing, Kleene, Markov and others and prove their equivalence. We consider subclasses of this class, for instance the class of *elementary functions* and the class of *primitive recursive functions*. We study *acceptable sets*, Kleene's *Recursion Theorems*, and the *Kleen-Mostowski-hierarchy*. We consider *Post's problem* and its solution by Friedberg and Mucnik. Finally, we consider Gödel's *Incompleteness Theorems.*

Literature
- S.C. Kleene, *Introduction to Metamathematics*, North Holland Publ. Co., Amsterdam 1952
- H. Rogers, *Theory of Recursive Functions and Effective Computability*, Mc Graw Hill, 1967
- G. Boolos, R. Jeffrey, *Computability and Logic*, Cambridge UP, 1974
- N.J. Cutland, Computability, *an Introduction to Recursive Function Theory,* Cambridge University Press, 1980
- P.G. Odifreddi, *Classical Recursion Theory*, Vol. I and II, Elsevier, Amsterdam, 1989, 1999

Examination
After having completed and submitted a number of exercises, students have to pass an oral examination.

Undergraduate algebraic geometry

Course ID: **WM031B** *6 ec* prof. dr. J.H.M. Steenbrink

Website
www.ru.nl/wiskunde/wiskunde/wie_wat_waar/steenbrink_jozef

Teaching methods
I intend to teach this course as a course of 6 ec. It is suitable for students in their third year as a choice subject and for students in the Master program who want to specialise in either Symbolic Computing, Mathematical Physics or Mathematics and Education.

Contents
This is an experimental course, based on reading the book 'Undergraduate Algebraic Geometry' by Miles Reid. It deals with polynomial equations in two and three variables (and a bit of general theory) focussing on conics (the case of degree two) and cubic curves and surfaces. By studying these particular cases and developing some tools to study them you may acquire a bit of the flavour of this beautiful subject!

Literature
'Undergraduate Algebraic Geometry' by Miles Reid, London Mathematical Society Student Texts 12, Cambridge University Press 1988

Examination
Oral

6 List of lecturers

Name	Email	Telephone	Room
Bosma, Dr. W.	w.bosma@math.ru.nl	52311	HG 03.716
Clauwens, Dr. F.J.B.J.	f.clauwens@math.ru.nl	52996	HG 03.713
Essen, Dr. A.R.P. van den	a.vandenessen@math.ru.nl	52993	HG 03.715
Gehrke, Prof. dr. M	m.gehrke@math.ru.nl	53220	HG 03.720
Hawkins, Dr. E.J.	e.hawkins@math.ru.nl		
Heckman, Prof. dr. G.J.	g.heckman@math.ru.nl	53233	HG 03.737
Hendriks, Dr. H.W.M.	harrie.hendriks@math.ru.nl	52868	HG 03.709
Kane, Dr. B.R.	b.kane@math.ru.nl	52873	HG 03.739
Kappen, Prof. dr. H.J.	b.kappen@science.ru.nl	14241	0.12 M244
Keune, Prof. dr. F.J.	f.keune@math.ru.nl	53230	HG 03.718
Koelink, Prof. dr. H.T.	e.koelink@math.ru.nl	52597	HG 03.742
Landsman, Prof. dr. N.P.	np.landsman@math.ru.nl	52874	HG 03.740
Maassen, Dr. J.D.M.	h.maassen@math.ru.nl	52991	HG 03.710
Muger, Dr. M.H.A.H.	m.mueger@math.ru.nl	52992	HG 03.744
Seevinck, Drs. M.P.			
Souvignier, Dr. B.D.	b.souvignier@math.ru.nl	53225	HG 03.717
Steenbrink, Prof. dr. J.H.M.	j.steenbrink@math.ru.nl	53144	HG 03.712
Suijlekom, Dr. W.D. van	waltervs@math.ru.nl	52873	HG 03.739
Veldman, Dr. W.H.M.	w.veldman@math.ru.nl	52972	HG 03.714
Wiegerinck, Dr. W.A.J.J.	w.wiegerinck@science.ru.nl	15040	0.24 M244

Index of courses