Faculty of Science

Prospectus 2007 - 2008

Mathematics

Master

Radboud University Nijmegen

Preface

This booklet is the propectus for the Master program of Mathematics at the Radboud University Nijmegen. It contains information about the objectives, the goals and the contents of the program.

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This prospectus has been made with great care. However the authors are not responsible for inaccuracies. If you have comments or proposals for improvements don't hesitate to contact them.

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1 Introduction

1.1 Structure

The Radboud University Nijmegen offers a Master of Science program in Mathematics. The English taught MSc program takes two years and provides students with a thorough knowledge of the relevant mathematics. The first year consists of core courses and electives in mathematics. The second year is largely devoted to the final thesis work, which involves participating in one of the department's advanced research projects or a traineeship or an internship within a company. If you pass the MSc exam you are awarded 'Master of Science'.

The Radboud University Nijmegen is a general university, offering almost all possible academic Programs, ranging from Arts and Law, to Medicine and Science. This Master program allows a substantial choice of topics from all these areas, thereby offering the possibility to combine Mathematics with other studies.

1.2 Admittance

In possession of a bachelor degree in Mathematics, one can of course enter the Mathematics Masterfase. In fact, a bachelor degree in Mathematics from any Dutch university will suffice. With an equivalent degree one can also be admitted, but approval of the Exam Committee Mathematics will be required. In this case, the Exam Committee can demand that one adds specific courses to the master program, depending on the Master specialization.
Eg: With a bachelor degree in Physics with a minor in Mathematics, one will be admitted to the Mathematics Master with the specialization Physics.

1.3 Credit point system

The Radboud University uses the European Credit Transfer System (ec) employed by all universities in the European Union. One year consists of 60 ec credits, the total Master program consists of 120 ec.

1.4 Dutch Master Program in Mathematics

The Departments of Mathematics of the Dutch universities have coordinated their efforts to enhance their Master Programs in Mathematics. Part of the cooperation is aimed at organizing joint courses in mathematics. The joint courses offer the students the highest quality of instruction and open opportunities for interaction with students of other institutes of mathematics. For students who intend to pursue a PhD program after completion of their Master Program the joint program may widen the range options for continuing their studies. For more information concerning the Dutch Master Program in Mathematics see the web site www.mastermath.nl .
Each master student in mathematics is obliged to attend courses from the Dutch Master Program in Mathematics with a total weight of 30 ec or more.

1.5 Mathematics clusters

Mathematics clusters form a new part of the Dutch mathematical landscape. A cluster is a collaboration between a number of Dutch universities, which is organized around a specific research area. Nijmegen takes part in two of the three clusters that are currently active, viz. DIAMANT (Discrete, Interactive & Algorithmic Mathematics, Algebra & Number Theory - involving CWI, RU, TU/e, UL) and CQT (Geometry and Quantum Theory, consisting of RU, UvA, UU). The RU does not take part in the third cluster NDNS (Nonlinear Dynamics of Natural Systems: CWI, RUG, UL, UU, VU).

While primarily founded in order to boost mathematical research in the Netherlands, the clusters are of great importance to master's students. This is firstly because they organize the bulk of the courses within the Dutch Master Program in Mathematics, and secondly because in writing your master's theses you will probably be attracted by some of the research themes offered by one of the clusters that Nijmegen takes part in.

The relevant websites are:
DIAMANT: http://www.win.tue.nl/diamant/
CQT: http://www.math.ru.nl/~landsman/cqt

1.6 Master thesis

At the start of the Master program, the student is expected to contact one or more prospective Master thesis supervisors to discuss a program. Normally the student will contact a thesis supervisor in the first semester of the Master program. The courses that are needed to prepare for the Master thesis work are determined by the Master thesis supervisor and the student together. It is therefore advisable to contact the prospective Master thesis supervisor to discuss the content of these courses. The individual program needs approval by the examination committee.

To select a prospective Master thesis supervisor, please look at the descriptions of the different specialisations within the department of mathematics. The contact persons of the departments can be approached at any stage for information or to set up a program in the electives section.

2 Program

2.1 Variants

The Master program at the Faculty of Science is offered in four variants: a research (O) variant, a communication (C) variant, an education (E) variant, and a business and management (MA) variant. At this moment, only the research variant has a complete program in the English language. The other variants are primarily aimed at the Dutch market and the Dutch educational system, and are therefore taught in Dutch.

- The O-variant leads students to a high level of knowledge in mathematics. It consists of advanced courses and a final research project including a Master thesis and an oral presentation of it. Students with this MSc in Mathematics are admissible to a PhD program. The program is suited as preparation for an academic career, in particular via a subsequent PhD study, but also for a career as mathematical researcher outside the universities.
- The C-variant is intended for a job in science communication in a broad sense. The program prepares students for a career in popularisation of science.
- The E-variant is intended as a preparation for a job in teaching mathematics and mathematics curriculum development.
- The MA-variant is intended as a preparation for jobs in the field of management. It prepares students for a career in science-related business and administration and for innovation and enterprise from a mathematical perspective.

In each variant the student chooses a specialisation within mathematics. The specialisation consists of a research project in one of the mathematics research groups of the Faculty of Science, including a Master thesis and an oral presentation and 30 ec of advanced courses to be selected in correspondence with the research topic.

2.2 Regulations Graduation project

The Graduation project is the completion to the Master Program. The central part consists of either a thesis or an apprenticeship. In both cases there is a staff member to supervise. In this paragraph one can find a more elaborate description of the Graduation Project.

Graduation Project including master thesis

The project consists of:

1. Acquiring knowledge about a specific subject by way of literature study, consultation and/or participating in a seminar.
2. Conducting scientific research.
3. Writing a thesis.
4. Presenting the master thesis and defending it in front of an audience of experts.

Graduation Project including an apprenticeship
The project consists of:
1. Acquiring knowledge about a specific subject by way of literature study, consultation and/or participating in a seminar
2. Doing work experience during an apprenticeship.
3. Writing an apprenticeship report.
4. Presenting the apprenticeship report and defending it in front of an audience of experts.

Assessment
The procedure for graduation project assessment consists of the following steps:
1. The supervisor approves of the thesis and /or the apprenticeship report, and notifies the program coordinator.
2. The program coordinator appoints a second rater, the supervisor organizes the presentation of the master thesis or the apprenticeship report in consultation with the student and the second rater.
3. After the presentation the supervisor determines the grade for the graduation project in consultation with the second rater.

2.3 The research variant (O-variant)

The program for the O-variant can be found in the table below. The amount of ec points is given between parentheses.

O-variant	
O-package	(30)
Mathematical Electives	(17)
Electives	(24)
Master Thesis Project	(40)
Philosophy	(3)
Free Electives	(6)

2.4 The communication variant (C-variant)

The communication variant (C) educates people for the areas of scientific communication (research, applications, media). The student who graduates in Communication is more than a beta (science) student, and has acquired complementary theoretical insights and communicative skills that broaden his own field of expertise (beta-gamma-integration). He has an insight in communication about innovation processes and processes of change, as well as an insight in the use of (mass) media and popularization.

Masterfase
In this main subject in the Masterfase 63 ec are contributed to the beta education, and 57 ec to the modules and apprenticeship of the C-variant.

C-variant	
Mathematical Specialisation	(30)
Mathematical Electives	(24)
C-package	(27)
Master Thesis Project	(30)
Philosophy	(3)
Free Electives	(6)

Courses
In the *first year* of the masterfase the following modules are offered:
- An introduction to Mass communication (3 ec)
- Communication & Change (3 ec)
- Crisis -and Risk communication (3 ec)
- Free Electives (6 ec)

In the *second year* of the masterfase the following modules are offered:
- Frames & images (3 ec)
- Contextual knowledge (3 ec)
- Paradigm's in perspective (3 ec)
- Strategies in Scientific Communication (3ec)
- Apprenticeship and & Master thesis (30 ec)

Free Electives
The free elective space can be filled with courses lectured by the Communication faculty. The approval for other free electives (for instance from Communication sciences, Psychology, Sociology, Biomedical Sciences, or other universities) will be given in consultancy with the student.

Apprenticeship & Master thesis. (24 ec + 6 ec)
The apprenticeship (24 ec) is communication research regarding the interface between Science and Society. Topics can be for instance: the role of scientists (experts) in conflicts between interest groups, conceptualization of social and scientific topics in the mass media (internet, film, tv. etc.), risk communication concerning disasters, the making of a Strategical Communication plan, the effectiveness of newsletters and websites, and political and social opinion forming.

The topic and methodology are always determined in consultation with the student, depending on his wishes and foreknowledge. The apprenticeship can be placed external (within a non-profit organization, ministry, or commercial organization), or accommodated on the faculty. The master thesis (6 ec) considers the results from the communication research in context with relevant literature, and makes recommendations for further research.

The quality and intensity if the supervision regarding the student will be guaranteed through apprenticeship agreements made with the internal and external supervisors.

2.5 The educational variant (E-variant)

E-variant	
Mathematical Specialisation	(30)
Mathematical Electives	(14)
E-traineeships	(57)
Master Thesis Project	(10)
Philosophy	(3)
Free Electives	(6)

The educational variant has a major course of 30 ec, in with preferably subjects are integrated that make it possible to view school math from a higher point of view, for instance because of the historical importance of the development of Mathematics.(the Master specialization Mathematics and education). Furthermore 14 ec free electives math, obligatory 3 ec Philosophy and 6 ec free electives. The course introduction to math didactics (8 ec), with is linked to supervising pupils in the project Mathematical Thinking Processes, can be taken in the fourth year, and can be included in the career preparation. By then, the study will be wound up in the fifth year with a final report of the career preparation at the ILS and a didactic thesis (10ec), if possible, both in mutual coherence.

2.6 The Management and Applications variant (MA-Variant)

MA-variant	
Mathematical Specialisation	(30)
Mathematical Electives	(24)
MT-package	(30)
Master Thesis Project	(27)
Philosophy	(3)
Free Electives	(6)

Goal

The set-up and filling-in of the management and applications variant strives to integration of beta and gamma aspects. In the MA variant the appliance of beta knowledge is the central aspect. Students are required to be capable of ding beta research in an applied setting; that also includes the handling of the company and managements difficulties, which cohere with those appliances.

For the filling-in of the MA variant thoughts go out to a mutual consensus about the contents of scientific professional education (in particular research apprenticeships) in the fourth year and the external apprenticeship location (organization that applies such beta know-how) in the fifth year.

Masterfase

In this main subject in the Masterfase 63 ec are contributed to the beta education, and 57 ec to the modules and apprenticeship of the MA-variant.

Courses

This year the following courses are being offered:

- *MA- courses* business and society (5ec), Organization knowledge (ec).
- *MA-integration courses:* Innovation management (5ec), Strategy and Marketing (5 ec), Financial Economic Management (5ec).
- *MA-free electives (*5ec*, choices of)*: Research Strategy and Management (3ec), Knowledge and entrepreneurship (3ec), Industrial Fine Chemistry, General Management skills.

Master project

The master project consists of conducting research in the cutting edge of science, technology, society and organization, mostly within a profit- or non-profit organization or the government.

Together with the student projects and research questions will be actively looked for, in which not only company or management knowledge is required, but also a science/ physics background is an advantage. Also it is desirable that a connection can be made between the fourth' year beta research and the master project MA. We emphasize the expression master project and not apprenticeship, because the student isn't supposed to just work at a company or institution, but, based on a completed phrasing of a question, handles a problem that is relevant to the organization.

Possible research areas for master projects are:

- Developing and/or evaluating of instruments for innovation-, science-, and environment policy.
- Researching the bottlenecks in the implementation of a technological innovation in an organization.
- Researching the consequences of actions of (for instance) environmentalists on company policy.
- Developing and/or evaluating instruments for personnel policy of a R&D department.
- Evaluation of the decision process of research projects in a R&D department.
- Developing instruments for improving the collaboration between universities and businesses.
- Developing instruments for supporting produce development.

3 Specialisations

3.1 Master specialisation

Mathematical research in the Radboud University Nijmegen is focussed on the following themes:

- Symbolic Computing
- Mathematical Physics
- Mathematics and Education

In any variant (O, C, E, MA) it is important to find a specialisation for graduation and, accordingly, a matching program of courses. Electives can be chosen freely, but the total package has to be approved by the examination board.

3.2 Symbolic Computing

The aim of the Master program *Symbolic Computing* is to explore areas of mathematics in which objects can be represented in an 'exact' way and are manipulated by algorithmic methods, often using a computer to actually perform the calculations. The term 'exact' is used to distinguish this discipline from fields like numerical analysis in which one usually has to be satisfied with (close) approximations to solutions. Typical topics of symbolic computing can be found in basically all branches of algebra but also in the parts of computer science dealing with formalized mathematics.

The application of symbolic computing to algebraic problems is the realm of computer algebra. Algebra comprises subjects like linear algebra, group and ring theory, number theory and commutative algebra, but there are also strong algebraic aspects to geometry and the theory of differential equations. The master program provides knowledge about important algorithms in these areas and about the concrete realization of their objects, emphasizing the common principles and showing the main applications. Students will learn how to compute efficiently with groups, number fields, algebraic curves, and how to solve systems of polynomial equations and certain differential equations. Experience with a variety of examples will result in a good understanding of the theoretical possibilities and practical limitations of algebraic algorithms.

The strength of the symbolic computing approach lies in the fact that algorithms not only deal with simple objects like numbers or strings but that the full hierarchy of algebraic objects is reflected in corresponding explicit data structures allowing to compute with these objects. For example, a permutation can be seen as a list of integers, a set of permutations generates a group and the set of subgroups forms a lattice with inclusion as incidence relation. Thus, we have four levels of objects (integers, permutations, groups and lattices of groups) which can all be directly manipulated by algorithms without having to retreat to a lower level.

Besides topics from pure mathematics, computer algebra also plays an important part in application domains like cryptography or communication networks. For example, more complex algebraic objects (like elliptic curves or high-dimensional lattices) are used to increase the security of encryption methods while at the same time new methods to attack encoded messages are explored using algorithms from computer algebra.

The areas of computer science in which symbolic computing plays an important role are concerned with questions of proving correctness of algorithms and implementations and with automatic theorem proving. The necessity to formalize algorithms rigorously in order to prove their correctness enforces a deep analysis of the structures involved in the algorithm and is therefore at the basis of truly understanding computational methods. On the application side, software certification is an area of rapidly growing importance, since more and more aspects of our daily life depend on computer chips and programs and one aims at 100% reliability for critical areas.

A fruitful interaction between computer algebra and formalized mathematics is found in exploring the relevance of methods from computer algebra in theorem-proving strategies as well as applying software certification techniques to computer algebra systems.

The fact that methods for symbolic computing are often implemented in large software packages (e.g. the computer algebra systems MAGMA or MAPLE) gives a further facet to the master program. A student will almost certainly implement new algorithms in an existing system and will thereby gather experience in handling and extending large software packages. Moreover, the design of data structures and algorithms for complex algebraic objects provides a hands-on approach to object-oriented programming and software engineering techniques.

Taking up a more general point of view, the process to transform an abstract algebraic question into a problem that may be approached algorithmically requires several important and valuable capabilities. Firstly, the algebraic objects have to be represented in a way accessible to a computer, secondly the path to a solution has to be split into several steps which can be turned into explicit algorithms and finally the results of a computation have to be interpreted in the context of the original abstract problem. Training these translation processes from abstract problems to explicit computations and backwards provides a student with capabilities which provide a profound basis for general problem solving tasks, thus giving qualifications which are sought for in many professional areas ranging from scientific and technical to communication and management positions.

The areas of expertise of the members of the Master program center around algorithmic methods in the following fields which would therefore be typical areas for research projects:
W. Bosma: computational number theory, algebraic curves, factorization methods, cryptography.
B. Souvignier: computational group theory, local fields, mathematical crystallography.
H. Barendregt: proof checking, software certification, automatic theorem proving.

The following courses are example of courses that will be given in the future and indicate some of the possible directions in which a student may work towards a master thesis. However, due to the broad spectrum of topics, it is possible to adapt the program to specific desires and students are welcome to suggest further fields of interest.

- Symbolic integration
- Computational number theory and cryptography
- Computational group theory
- Chance as a method: Randomized algorithms

The following courses may be taught in different forms: Either as a specialized course for the Master program 'Symbolic Computing' with a value of 10 ec, or in a slightly softened version as an orienting course in the Master phase. In the latter case they will be assigned a value of 6 ec and are also suitable as optional courses for the 3rd year.

- Coding theory
- Graph theory
- Chance as a method: randomized algorithms
- Lattices and crystallographic groups

3.3 Mathematical Physics

From the time of Newton (1642-1727) until about 1930, mathematics and theoretical physics were inseparable. Breakthroughs typically took place simultaneously in both areas, and progress at both fronts often even resulted from the work of a single scientist, such as Newton himself, Huygens, Euler, Lagrange, Laplace, Fourier, Gauss, Poisson, Cauchy, Jacobi, Hamilton, Riemann, and Poincaré. Their work provided the foundations of 'classical' mathematical physics (as well as of large areas of mathematics), which culminates in the field of partial differential equations (the Maxwell equations are a case in point). A second stage in the development of mathematical physics is connected with some of the greatest names in 20th century science, like Einstein, Born, Dirac and Wigner on the physics side, and Hilbert, Weyl, von Neumann and Kolmogorov on the maths side. The cross-fertilization of mathematics and physics led by these people was instrumental in establishing key areas of modern physics like general relativity and quantum mechanics as well as parts of mathematics like differential geometry, Lie groups and functional analysis.

This typical cross-fertilization subsided between about 1930-1975, when research at the frontiers of physics felt no need for advanced or new mathematics (whose relevance to physics was even openly derided by Feynman), whilst simultaneously mathematics began to be developed according to its own internal criteria established by Hilbert and others (notably the French Bourbaki group). (In addition, after 1945 some of the greatest mathematicians like Grothendieck refused to make use of insights from modern physics because of its connection to nuclear weapons.) This has led, for example, to the creation of modern algebraic geometry and algebraic number theory by Weil, Grothendieck, Serre, Deligne, and others. This development may be said to have culminated in the extremely deep and abstract proof of Fermat's Last Theorem by Wiles in the mid-1990s. From about 1975, however, mathematical physics has began to regain the élan it used to have.

First, mathematicians like Atiyah, Singer and Penrose, and physicists like 't Hooft and Witten recognized the connection between differential geometry and gauge theories. This connection goes via the notion of index theory (originating in analysis) and is crucial, for example, in the technical implementation of Sacharov's scenario for baryogenesis mentioned earlier. This recognition has led to very important progress in both physics (magnetic monopoles, instantons, anomalies, and other topological phenomena in classical and quantum field theory) and mathematics (e.g. Donaldson theory and Floer homology). In its immediate wake, deep relationships between algebraic geometry and quantum field theory and string theory were discovered and developed by Witten, Kontsevich and others. The work of Dijkgraaf, Verlinde and Verlinde also played an important role here.

Second, Connes began to develop an entire new domain of mathematics called noncommutative geometry on the basis of ideas from quantum physics (e.g. the Dirac equation), operator algebras (an area of mathematics created by von Neumann in the 1930's) and index theory (the field launched by Atiyah and Singer just mentioned). This body of work has led to breakthroughs in a number of areas in pure mathematics (like index theory and foliation theory). Furthermore, as might have been expected, the subject was successfully applied to physics within a decade after its inception, for example to the quantum Hall effect, the theory of quasicrystals, and the Standard Model of elementary particle physics. More recently, noncommutative geometry has also been related to renormalization theory in perturbative quantum field theory, and to string theory.

Third, the classical area of integrable systems (going back to Lagrange, Jacobi and others, with important later contributions by Lax) underwent a complete rejuvenation in that it got related to the Langlands program (originally an area of pure mathematics in which number theory and representation theory interacted). Also, the notion of a Frobenius manifolds emerged from the work of Witten, Manin, Dijkgraaf and others as a new setting for integrable systems. These three areas together have culminated in an independent field of research called the geometric Langlands program.

The above developments have been widely recognized by the mathematical community. Atiyah has been awarded both of the two most prestigious prizes in mathematics, viz. the Abel prize (with Singer) and the Fields Medal. Connes and Kontsevich won the Fields Medal as well, as did Witten (although he is a physicist). Lax was awarded the Abel Prize. Penrose has won the Wolf Prize and numerous other awards, as did 't Hooft. And so on and so forth.

Research at Nijmegen

Each of the three current research directions in mathematical physics just mentioned is well represented at Nijmegen, and there are other themes as well, so students interested in research in mathematical physics are offered a rich choice. Clauwens' research is in algebraic topology. Heckman's research lies in the interaction between Lie theory, integrable systems, and geometry, and is closely related to the third topic above. It is currently centered around the link between the geometric Langlands program and Hitchin's integrable system. Landsman's research combines noncommutative geometry with quantization theory (i.e. the theory that tries to establish the precise mathematical relationship between classical and quantum physics). One goal is the quantization of singular spaces, in the hope of eventually developing a quantum theory of the Big Bang (a purely classical notion which according to Stephen Hawking and others is probably smoothened out in quantum theory). Maassen works in quantum probability, including the application of stochastic calculus to the interaction of molecules with light, and the functional analysis of quantum noise. In addition he works in the modern theory of quantum information and quantum computing. Müger specializes in category theory, operator algebras and their applications to quantum field theory. He is also interested in constructive quantum field theory. Steenbrink's work is mainly in algebraic geometry. Over the last few years it has focused on two streams: the study of discriminant complements and moduli spaces, and the study of threefolds which are double covers of projective space (double solids).

Preparation

When coming from the BSc Program in mathematics, the student is recommended to prepare for the MSc Program by filling in the free space of the BSc Program in the way described in the Bachelor Prospectus. This is as follows:

- 1st semester: Mechanics 1 & 2 (6 ec), Laboratory course in Physics 1a (1 ec)
- 2nd semester: Mechanical waves(2 ec), Introduction to quantum mechanics (3 ec)
- 3d semester: Analytical mechanics (3 ec), Vibrations and waves (3 ec)
- 4th semester: Quantum mechanics 1a & 1b (6 ec), Electricity and magnetism 1 & 2 (6 ec) Introduction Fourier theory (3 ec), Complex functions (3 ec)
- 6th semester: Introduction to partial differential equations (6 ec), Analytic geometry (3 ec)

Special relativity (3 ec) is highly recommended as well. Before entering the master program in Mathematical Physics, one is recommended to take Quantum mechanics 2 (5 ec) and Quantum mechanics 3 (6 ec). Otherwise, one has to do these courses during the master itself.

Master program

The following courses, to be taken during the fourth and fifth years, comprise the actual Master Program.

From mathematics:

- Hilbert space and quantum mechanics (6 ec)
- Group theory (6 ec)

One or two items chosen from:
- Probability theory (6 ec)
- Stochastic analysis (6 ec
- Stochastic processes (6 ec)

From physics:

- Classical electrodynamics (6 ec)
- Statistical mechanics (6 ec)
- Quantum mechanics (6 ec)

Courses offered on demand (sometimes as reading groups) are:

- Quantum theory of condensed matter (6 ec)
- Infinite dimensional Lie algebras (6 ec)
- Quantum probability theory (6 ec)
- Constructive quantum field theory (6 ec)

3.4 Mathematics and Education

This program combines mathematics that is especially useful for future mathematics teachers with the E-traineeships of the Instituut voor Leraar en School (ILS, Institute for Teacher and School). There are various routes to become a mathematics teacher. There is a direct route of one year mathematics and one year teacher training and a longer route that also involves a half year communication. In any case the idea is to combine mathematics and education as much as possible.

In the mathematics part there will be courses which are linked to school mathematics, have interesting historical contexts and which require an active participation of the students. Parts of the teacher training program will be combined with activities within the Mathematics department: the Dutch Kangaroo competition, the Ratio project for school curriculum development and the Wiskundig Denken (Mathematical thinking) project for pupils in the last years of secundary school.

Further information: Prof.dr. F.J. Keune.

4 Dutch Master Program in Mathematics

4.1 Program and schedule

In this chapter you find a list of all Master courses offered in 2007/2008 in the framework of the **DUTCH MASTER PROGRAM IN MATHEMATICS.** For descriptions of these courses and further details see: http://www.mastermath.nl .

You have to register for these courses at: http://www.mastermath.nl/registration/ .
Abbreviations:

- (LNMB)= these courses are organized by the Dutch Network on the Mathematics of Operations Research
- (MRI)= these courses are organized by the Mathematics Research Institute and are recommended only for advanced students who specialize in dynamics of differential equations.
- (3TU)= these courses are part of a joint MSc program in Applied Mathematics of the 3 Dutch technical universities (Technische Universiteit Eindhoven, Universiteit Twente and Technische Universiteit Delft)

4.2 Course Schedule Fall 2007

Courses start **Tuesday** September 11, 2007 and end December 21, 2007.

Monday	**Universiteit Utrecht (LNMB & TU+)**
10:15 - 12:00 hrs:	Discrete Optimization (6 cp)
Instructors:	Haemers, W. (Universiteit van Tilburg)
10:15 - 14:45 hrs:	Introduction to Stochastic Processes (4 cp)
Instructors:	Litvak, N. (Universiteit Twente) Scheinhardt, W.R.W. (Universiteit Twente)
Venue:	This is a crash course. Meetings only on Mondays and Tuesdays September 3, 4, 10 and 11, 2007
13:00 - 14:45 hrs	Continuous Optimization (6 cp)
Instructors:	Still, G. (Universiteit Twente)

Monday	**Universiteit Utrecht / Universiteit Twente**
10:15 - 12:00 hrs:	Systems and Control (6 cp)
Instructors:	Polderman, J.W. (Universiteit Twente) Trentelman, H.L. (Rijksuniversiteit Groningen) Stoorvogel, A.A. (Universiteit Twente)
Venue:	Extra classes at Universiteit Utrecht: September 17, 2007 October 1, 2007 October 15, 2007 November 12, 2007 Intensive week at Universiteit Twente: October 29 - November 2, 2007
13:00 - 14:45 hrs:	Nonlinear Systems Theory (6 cp)
Instructors:	Schaft, A.J. van der (Rijksuniversiteit Groningen) Scherpen, J.M.A. (Rijksuniversiteit Groningen)
Venue:	Extra classes at Universiteit Utrecht: September 17, 2007 October 1, 2007 October 15, 2007 November 12, 2007 Intensive week at Universiteit Twente: October 29 - November 2, 2007

Monday	**Universiteit Utrecht (LNMB)**
15:00 - 16:45 hrs:	Simulation (6 cp)
Instructors:	Ridder, A.A.N. (Vrije Universiteit) Tijms, H.C. (Vrije Universiteit)

Tuesday	**Universiteit Utrecht**
10:15 - 13:00 hrs:	Permutation Groups and Coxeter Groups (8 cp)
Instructors:	Cohen, A.M. (Technische Universiteit Eindhoven) Lenstra, H.W. (Universiteit Leiden)
14:00 - 16:45 hrs:	Numerical Linear Algebra (8 cp)
Instructors:	Gijzen, M.B. van (Technische Universiteit Delft) Sleijpen, G.L.G. (Universiteit Utrecht)
14:00 - 16:45 hrs:	Diophantine Approximation (8 cp)
Instructors:	Evertse, J.H. (Universiteit Leiden) Beukers, F. (Universiteit Utrecht)

Tuesday	**Universiteit van Amsterdam**
10:15 - 13:00 hrs:	Functional Analysis (8 cp)
Instructors:	Jeu, M.F.E. de (Universiteit Leiden) Ran, A.C.M. (Vrije Universiteit)
14:00 - 16:45 hrs:	Partial Differential Equations (8 cp)
Instructors:	Vorst, R.C.A.M. van der (Vrije Universiteit)

Wednesday	**Vrije Universiteit**
10:15 - 13:00 hrs:	Measure Theoretic Probability (8 cp)
Instructors:	Spreij, P.J.C. (Universiteit van Amsterdam)
14:00 - 16:45 hrs:	Asymptotic Statistics (8 cp)
Instructors:	Jongbloed, G. (Technische Universiteit Delft)

Wednesday	**Universiteit Utrecht**
10:15 - 13:00 hrs:	Quantum Groups and Knot Theory (8 cp)
Instructors:	Stokman, J.V. (Universiteit van Amsterdam) Cohen, A.M. (Technische Universiteit Eindhoven) Opdam, E.M. (Universiteit van Amsterdam)
14:00 - 16:45 hrs:	Semi-simple and Affine Lie Algebras (8 cp)
Instructors:	Ban, E.P. van den (Universiteit Utrecht) Leur, J.W. van de (Universiteit Utrecht) Helminck, G.F. (Universiteit Twente)

4.3 Course Schedule Spring 2008

Courses start Monday February 4, 2008 and end May 23, 2008.

Monday	**Universiteit Utrecht**
10:15 - 12:00 hrs:	Advanced Linear Programming (6 cp)
Instructors:	Pendavingh, R. (Technische Universiteit Eindhoven)
10:15 - 12:00 hrs:	Applied Statistics (6 cp)
Instructors:	Bucchianico, A. di (Technische Universiteit Eindhoven)
13:00 - 14:45 hrs:	Scheduling (6 cp)
Instructors:	Hurink, J.L. (Universiteit Twente)
13:00 - 14:45 hrs:	Stochastic Differential Equations (6 cp)
Instructors:	Weide, J.A.M. van der (Technische Universiteit Delft)
15:00 - 16:45 hrs:	Queueing Theory (6 cp)
Instructors:	Adan, I.J.B.F. (Technische Universiteit Eindhoven) Resing, J.A.C. (Technische Universiteit Eindhoven)

Monday	**Universiteit Utrecht / Universiteit Twente**
10:15 - 12:00 hrs:	Advanced Modelling in Science (6 cp)
Instructors:	Heemink, A.W. (Technische Universiteit Delft)
Venue:	This is an intensive course. See Courses and Exams for more information about the organization of intensive courses. First meetings will be in Utrecht, the intensive week will be in Twente (Enschede).
13:00 - 14:45 hrs:	Applied Finite Elements (6 cp)
Instructors:	Segal, A. (Technische Universiteit Delft) Vegt, J.J.W. van der (Universiteit Twente)
Venue:	This is an intensive course. See Courses and Exams for more information about the organization of intensive courses. First meetings will be in Utrecht, the intensive week will be in Twente (Enschede).

Tuesday	**Vrije Universiteit**
10:15 - 13:00 hrs:	Elliptic Curves (8 cp)
Instructors:	Cornelissen, G.L.M. (Universiteit Utrecht) Stevenhagen, P. (Universiteit Leiden)
14:00 - 16:45 hrs:	Cryptology (8 cp)
Instructors:	Cramer, R.J.F. (CWI) (Universiteit Leiden) Schoenmakers, L.A.M. (Technische Universiteit Eindhoven) Fehr, S. (CWI)

Tuesday	**Universiteit Utrecht**
10:15 - 13:00 hrs:	Mathematical Methods to Gain Biological Insights (8 cp)
Instructors:	Diekmann, O. (Universiteit Utrecht) Planqué, R. (Vrije Universiteit)
14:00 - 16:45 hrs:	Dynamical Systems Generated by Ordinary Differential Equations and Maps (8 cp)
Instructors:	Diekmann, O. (Universiteit Utrecht) Kuznetsov, Y. (Universiteit Utrecht)
14:00 - 16:45 hrs:	Numerical Methods for PDE (8 cp)
Instructors:	Stevenson, R.P. (Universiteit Utrecht)

Wednesday	**Universiteit Utrecht**
10:15 - 13:00 hrs:	Differential Geometry (8 cp)
Instructors:	Looijenga, E.J.N. (Universiteit Utrecht) Ban, E.P. van den (Universiteit Utrecht)
14:00 - 16:45 hrs:	Scattering Theory (8 cp)
Instructors:	Koelink, H.T. (Technische Universiteit Delft) Landsman, N.P. (Radboud Universiteit Nijmegen)

Wednesday	**Universiteit van Amsterdam**
10:15 - 13:00 hrs:	Stochastic Processes (8 cp)
Instructors:	Spieksma, F.M. (Universiteit Leiden)
14:00 - 16:45 hrs:	High Dimensional Data and Multivariate Analysis (8 cp)
Instructors:	Meulman, J.J. (Universiteit Leiden) Vaart, A.W. van der (Vrije Universiteit) Wiel, M.A. van der (Vrije Universiteit)

5 Description of courses

Algebraic curves and Riemann surfaces

Course id: **WM029A** *10 ec* Remark: This couse will not be given in 2007-2008. prof. dr. J.H.M. Steenbrink

Teaching methods
- 28 hrs lecture
- 28 hrs tutorial

Pre-requisites
Bachelor's degree in mathematics. Some complex function theory.

Learning outcomes
After this course the student is familiar with the notions of Riemann surface, holomorphic map, degree of a proper map. He is familiar with the examples of complex tori using theta functions. He knows several definitions of the germs of a compact Riemann surface, and the calculus of differential form on them, including the Zeuthen-Hurwitz formula and the residue theorem. He can deal with linear systems to map Riemann surfaces into projective spaces. He has seen several applications of the Riemann-Roch theorem.

Description
A Riemann surface is a connected space which is a union of open pieces, each of which is a disc in the complex plane, and which are glued together via holomorphic transformations. It makes sense to perform complex analysis on a Riemann surface, in a similar way as one may perform calculus on manifolds. The local theory is just the complex function theory of one variable, but global aspects are very important. Compact Riemann surfaces admit embeddings in complex projective spaces, and their images are then described by polynomial equations. This makes a link with algebraic geometry.
The course is based on Chapters I till VII included of Miranda's book with the same name. It is meant as a second course in the Master curriculum.

Literature
R. Miranda, *Algebraic Curves and Riemann Surfaces*. *Graduate Studies in Mathematics*, Volume 5, American Mathematical Society, 1995.

Examination
Oral

Algebraic K-Theory

Course id: **WM056B** *10 ec* prof. dr. F.J. Keune

Teaching methods

- 28 hrs lecture
- 28 hrs tutorial

Learning outcomes

The student has understanding of K_0, K_1 and K_2 as functors of rings. He can relate these functors to more classical concepts as modules, determinants and reciprocity laws.

Description

Finitely generated projective modules, especially over Dedekind domains. Stable equivalence of projective modules. The functor K_0. The general linear grouop and the functor K_1. The Steinberg group and the functor K_2. Exact sequences in K-theory. Steinberg symbols on a field, in particular on a number field.

Literature

J. Milnor, *Introduction to Algebraic K-theory*, Princeton University Press, 1971.
B. A. Magurn, *An Algebraic Introduction to K-Theory*, Cambridge University Press, 2002

Examination

To be arranged with the participants.

Algebraic number theory

Course id: **WM049B** *10 ec* prof. dr. F.J. Keune

Teaching methods
- 28 hrs lecture
- 28 hrs tutorial

Learning outcomes
The student has a level of understanding of algebraic number theory that is needed for the proof of the theorem of Kronecker and Weber on Abelian number fields. He/she can relate the theory to concrete number theory problems.

Description
Algebraic number theory originates from problems about Diophantine equations and its main subject is the structure of algebraic number fields, i.e. finite extensions of the rationals. The main goal of the course is to obtain understanding of the theorem of Kronecker and Weber: every Abelian number field is contained in a cyclotomic field. For this some knowledge of Galois theory is required, especially in the second half of the course. There will be special sessions for students who still have to learn the Galois theory needed for this course. There will be special emphasis on the computation of important invariants of number fields: ideal class groups, groups of units.

Literature
Lecture notes.

Examination
Participants are expected to give talks on various sections and on solutions of exercises. From each participant a final presentation is required.

Algebraic topology

Course id: **WM045A** *10 ec* dr. F.J.B.J. Clauwens

Teaching methods
- 28 hrs lecture
- 28 hrs tutorial

Learning outcomes
The student should be familiar with the notion of simplicial complex and be able to compute the homology of a polyhedron effectively in practical cases. He can list the properties of both simplicial and singular homology and can reproduce the derivation of these properties. He should also be able to explain important applications of these theories like the Lefschtez Fixed Point Theorem and the Generalized Jordan Curve Theorem.

Description
The course starts with the geometry of 'simplicial complexes'. These are spaces built from simplces (= line segments, triangles, tetrahedra etc.). From the way these simplces fit together we construct certain abelian groups, the so called 'simplicial homology groups'. The definition enables one to compute these groups effectively.

Subsequently we study what happens when a space is dissected differently into simplices. This results in the theorem that homeomorphic spaces have isomorphic homology groups. Thus sometimes spaces can be shown to be essentially different by computing their homology. As a further application we show how a continuous map from a polyhedron to itself gives rise to a 'Leftschetz number' in the integers with the property that it vanishes for a map without fixed point.

Next we discuss the singular homology groups of an arbitrary topological space. These can not be computed directly from their definition. We will list a number of properties of these groups the so called 'Eilenberg-Steenrod axioms'. These properties characterize the theory on polyhedra. We will also see how these properties can be used to compute these groups.
Finally we prove a high-dimensional generalization of the Jordan Curve theorem: consider a continous map from the unit sphere in Euclidean space to the same Euclidean space; then the complement of its image has two open components with that image as a common border.

Literature
J.R. Munkres: `*Elements of Algebraic Topology'*, Addison-Wesley Publ. Comp., 1984.

Examination
Oral.

Axiomatic set theory

Course id: **WM038A** *10 ec* Spring 2008 dr. W.H.M. Veldman

Teaching methods
- 28 hrs lecture
- 28 hrs tutorial

Learning outcomes
The student comes to know the story of the formalization of set theory and some of its heroic results and open questions.

Description
We explain how set theory started with *Cantor's diagonal* argument and his *continuum hypothesis*. We consider Zermelo's *Axiom of Choice* and some famous applications, including *Zorn's Lemma* and the *Banach-Tarski-paradox*. We then list the axioms given by Zermelo and Fraenkel and develop set theory from them including the theory of ordinals and cardinals. We go on to consider Gödel's *constructible sets* and his proof of the consistency of the continuum hypothesis. We try to obtain some idea of the *forcing method* developed by Cohen for proving the consistency of the negation of the continuum hypothesis. If time permits, we also discuss Mycielski's *Axiom of Determinacy*, and/or Aczel's *Anti-foundation axiom*.

Literature
- K. Kunen, *Set Theory, an Introduction to the Independence Proofs*, North Holland Publ. Co., Amsterdam, 1980.
- K. Devlin, *The Joy of Sets: an Introduction to Contemporary Set Theory*, Springer Verlag, New York, 1987
- T. Jech, *Set Theory*, Springer verlag, New York, 1997
- Y.N. Moschovakis, *Notes on Set Theory*, Springer Verlag, New York, 1994
- R.L. Vaught, *Set theory: an Introduction*, Birkhäuser, Boston, 1985

Examination
After having completed and submitted a number of exercises, students have to pass an oral examination.

Coding Theory

Course id: **WM005B** *6 ec* dr. W. Bosma

Teaching methods
- 28 hrs lecture
- 28 hrs tutorial

Description
Coding theory deals with error correcting codes. These are constructed in order to reconstruct digital messages in which some bits may have been corrupted (think of noise added during the transmission of satellite photos or errors while reading from a CD). Error correction is achieved by adding redundant information, but this causes contradicting effects: adding bits slows down the transfer rate while it enhances the capacity of correcting errors.

In this course we will deal with algebraic aspects of linear (block-)codes which may be described as (sub-)vector spaces over a finite field. Wellknown and much applied constructions (quadratic residue-, BCH-, Reed-Muller- and cyclic codes) will be discussed as well as some recent constructions using algebraic geometry.

The most important algebraic methods are provided by the theory of finite fields and their polynomial rings.

Literature
Syllabus by dr. R.H. Jeurissen

Cohomology and characteristic classes

Course id: **WM034B** *6 ec* dr. F.J.B.J. Clauwens

Teaching methods
- 28 hrs lecture
- 28 hrs tutorial

Pre-requisites
Algebraic Topology. It is also useful to have some experience with differentiable manifolds and/or vector bundles.

Subjects
- Differentiable manifolds
- Vector bundles
- Cohomology
- Thom isomorphism
- Stiefel-Whitney classes
- Chern classes
- Pontrjagin classes
- Thom spaces
- Transversality
- Bordism theory

Literature
John W. Milnor and James Stasheff: *'Characteristic Classes' Annals of mathematics studies 76,* Princeton University Press 1974 ISBN 0-691-08122-0

Commutative algebra

Course id: **WM026A** *10 ec* Fall dr. A.R.P. van den Essen

Teaching methods
- 28 hrs lecture
- 28 hrs tutorial

Learning outcomes
The student is acquainted with the theory of modules over commutative rings. In particular he is familiar with the theory of Noetherian and Artin modules, tensor products, localization and all basic notions of commutative algebra.

Description
What is commutative algebra about? To make this clear let's start with a k-vector space V, where k is a field. So V is a set equipped with an addition, which makes V into an abelian group, and a scalar multiplication with scalars from k. Furthermore the classical distibutive laws hold. If we replace k by an arbitrary commutative ring R we get a so-called R-module.

This notion generalises most of the notions one meets during a Bachelor's study Mathematics. For example it will turn out that a Z-module is the same as an abelian group, a $k[x]$-module is the same as a k-vector space together with a linear transformation and an ideal I in a ring R is an example of a so-called R-submodule of R. Also the quotient ring R/I is an R-module etc. The theory of R-modules is much more complicated than the theory of vector spaces; many problems are still unsolved. The general philosophy is that the 'nicer' the ring R is, the more we know about its R-modules. The language of modules is an indispensable tool in nowadays Mathematics. In this course we discuss the most fundamental concepts and results of modern commutative algebra. Many of the notions introduced in this course will also be used in various other courses.

If you are planning to specialize in algebraic geometry, algebraic topology, number theory, computer algebra or polynomial mappings, this course is a must.

Literature
We follow the excellent book *'Introduction to commutative algebra'* by M.F. Atiyah and I.G. MacDonald.

Examination
The student has to make a series of exercises.

Coxeter Groups

Course id: **WM051B** *6 ec* prof. dr. G.J. Heckman

Teaching methods
- 28 hrs lecture
- 28 hrs tutorial

Pre-requisites
Linear algebra, Symmetry.

Learning outcomes
The student acquires a basic knowledge on group presentations and its geometric meaning, with a good deal of emphasis on the beautiful example of Coxeter groups.

Description
Coxeter groups are groups with particular presentations coming from the geometry of polyhedral tessellations. This gives the subject both an algebraic and a (combinatorial and differential) geometric flavour. Coxeter groups show up naturally in various parts of mathematics, such as Lie theory (the Killing-Cartan classification of simple Lie groups), algebraic geometry (the period map of Enriques or K3 surfaces). Coxeter groups are also the mathematical frame work behind some of the geometric art of M.C.Escher.
In the course we will start with a basic introduction to Coxeter groups leading to the Tits theorem. Subsequently we will discuss Coxeter's paper on Escher's Circle Limit III and McMullen's paper on the link between Coxeter groups and the Hilbert metric.
The course is well suited for students from 'education' (especially the paper of Coxeter), 'symbolic computing' and 'mathematical physics' (for people interested in the simple Lie groups).

Literature
- J.E. Humphreys, *Reflection groups and Coxeter groups*, Cambridge Studies Adv Math 29, CUP, 1990.
- H.S.M. Coxeter, *The trigonometry of Escher's woodcut Circle Limit III*, Math Intellegencer, 1996.
- C. McMullen Coxeter groups, *Salem numbers and the Hilbert metric*, preprint 2002, available from his webpage: www.math.harvard.edu ... etc

Examination
Oral exam / written assignment.

Credit Risk

Course id: **WM021B** *6 ec* prof. dr. M.C.A. van Zuijlen

Teaching methods
- 28 hrs lecture
- 28 hrs tutorial

Pre-requisites
Probability Theory and some Statistics, Option Theory (arbitrage theory of contingent claim valuation), Stochastic Processes and Stochastic Calculus.

Learning outcomes
After having taken this course the student will know what credit risk management is about, and which principles are underlying it. He will be able to think sensibly about the topic and will be able to read additional literature on the subject. He will have seen a lot of techniques of credit risk modeling.

Description
The goal of this course is to make the student familiar with what credit risk is about, the basic principles of credit risk modelling and the influence of credit risk for contingent claim valuation. This course will cover some of the techniques used for credit risk. In particular, time will be spent on:
The modelling of the term structure of interest rates, definitions of credit risk and credit derivatives, definitions and discussion of the main notions in credit risk modelling (default probabilities, hazard rates, etc.), advanced mathematical modelling techniques, in particular notions related to discontinuities in stochastic processes, structural credit risk modelling, intensity-based credit risk modelling, dependence concepts relevant for credit risk modelling. If time permits we could shift focus to quantitative risk management of which credit risk management is a part.

Literature
Tomasz Bielecki, Marek Rutkowski, Credit Risk: *Modeling, Valuation and Hedging*, Springer Verlag, 2004.
Andrew Cairns, *Interest Rate Models An Introduction*, Princeton University Press, 2004.
David Lando, *Credit Risk Modeling*, Princeton University Press, 2004.
Alexander McNeil, Rüdiger Frey, Paul Embrechts, *Quantitative Risk Management,* Princeton University Press, 2005.
Philipp Schönbucher, *Credit Derivatives Pricing Models*, John Wiley & Sons, 2003.

Examination
Students will be graded based upon work that has to be handed in and (possibly) given presentations during the course. At the end of the course the student will have to write a paper about a publication in the field.

Differential topology

Course id: **WM035B** *6 ec* dr. M.H.A.H. Muger

Teaching methods

- 28 hrs lecture
- 28 hrs tutorial

Description

Differential topology is a branch of general topology focussing on a class of particularly nice spaces, that is smooth manifolds. The study of the latter on one hand leads to exciting new problems (e.g. exotic smooth structures), but on the other hand a solid understanding of smooth manifolds is also fundamental for many further developments, like the study of manifolds with riemannian, symplectic or Poisson structures as well as for analysis on manifolds and index theory. Besides the fundamentals we will also discuss some connections with algebraic topology (degree, Euler number). For a more detailed overview see the table of content of the syllabus (mainly chapter 2).

Literature

A. Syllabus provided by the lecturer.
B. Supplementary references:

- V. Guillemin and A. Pollack: 'Differential topology', Prentice-Hall Inc.
- Bröcker and Jänich: 'Introduction to differential topology', Cambridge University Press, 1982 (or the original version in German)
- M. Hirsch: ′Differential topology′, Springer Graduate Texts in Math 33

Elementary Number Theory

Course id: **WM055B** *8 ec* prof. dr. F.J. Keune

Teaching methods
- 40 hrs lecture

Pre-requisites
Knowledge of the rudiments of groups and rings, as they were discussed in the first weeks agebra courses in bachelor Math educations.

Learning outcomes
The student has knowledge of number theory, in particular reciprocity laws, with enable him to understand certain modern developments from an elementary point of view.

Description
This course is primarily intended for a Masterprogram Mathematics and and Education, in particular the education variant of a Master course Mathematics. It consists of two parts.The first part consists of lectures including excercises. Reciprocity laws are the main theme; the quadratic, kubic and biquadratic reciprocity laws.

A number of subjects will be raised: unequivocal disintegration with applications, the structure of remaining class figures of the integers, arithmetical functions, finite numbers, Gauss and Jacobi sums. The goal of the course is to offer an insight in modern developments in the number theory form from an elementary perspective. Students will be working in small groups on a number theoretical subject, presenting their results in the second half of the course and writing an introductory about it.

The course is based on the first nine chapters of the book of Ireland and Rosen.

Literature
Kenneth Ireland, Michael Rosen, *A Classical Introduction to Modern Number Theory, Graduate Texts in Mathematics*, Vol. 84, Springer-Verlag, ISBN 0-387-97329-X

Examination
Final assessment is based half on homework and half on a lecture about a subject of choice.

Extra information
40 hours lecture

Familiarisation traineeship Education

Course id: **FE0001B** *3 ec*

Teaching methods

- 60 hrs traineeship secondary education
- 20 hrs preparations/traineeship assignments/report

Learning outcomes

The 'Work experience E-orientation' offers students the opportunity to further familiarise themselves with the E(ducation)-orientation during the master phase (following the CEM-course as part of the bachelor phase).

Description

Planning: The traineeship at a secondary education institute does not only involve working alongside the teacher and observing, but also teaching a class yourself (8 lessons) and discussing these with the supervisory school teacher.

Experience shows that you will need to be at school 2 days per week for a period of 4 to 5 weeks in order to acquire the necessary experience. However, the student is free to make other arrangements in consultation with the traineeship school concerning some other schedule.
The schools offer two possible periods for the purpose of this work experience, namely from October 1st until December 1st and from February 1st until April 1st. This periods are generously scheduled in order to give the student and the school the opportunity to flexibly plan the traineeship in the course of the fourth year of one's studies.

Supervision: The university provides supervision in the form of a teaching methodologist from the Institute for Teachers and School (ILS). This institute teacher arranges an introductory meeting, maintains contacts with the schools, provides literature and assignments and assesses the report. The institute teacher visits the traineeship school once for consultations on location, supplemented with a lesson observation, if desired.
Secondary school teachers supervise at school.

Extra information

This work experience traineeship is not mandatory, but it is certainly advisable for everyone who wishes to be a fully-qualified teacher. The traineeship can be flexibly scheduled.
Contact: secretariat of the Institute for Teachers and School, Gymnasion, tel. 024-3530093 or 3530094.

The traineeship department of the ILS makes arrangements for trainee posts based on the applications pertaining to this particular form of traineeship. Please keep in mind that it may prove necessary to use a weekly rail pass.

Philosophy of mathematics

Course id: **WM040B** *3 ec* on request dr. W.H.M. Veldman

Teaching methods
- 30 hrs lecture

Learning outcomes
The student will learn to see that the question about the nature of mathematics is one of the most important questions in philosophy, and that meticulous mathematical thinking and philosophical contemplation can stimulate each other.

Description
During the course we discuss: Plato's Ideas and the place of the mathematical objects, Aristotel's view, Kant's view on the nature of mathematical statements, Frege and Russel's logicism, Russel's paradox, Cantor's discoveries, Brouwer's intuitionistic criticism, Goedel's incompleteness thesis, Goedel's platonism,Wittgenstein's thoughts.

Examination
The student studies various texts of choice, and is assessed about them orally.

Extra information
The course will be tought on request
The course can be taken to fill in the obligatory part of Philosophy 1
The students will be given home texts that will be discussed in the lectures, during which one will be able to ask questions, or have discussions.

Fundamental questions

Course id: **WB028B** *6 ec* dr. W.H.M. Veldman

Teaching methods

- 28 hrs lecture
- 28 hrs tutorial

Learning outcomes

The students will learn that questioning the nature of mathematics and the exact foundations of mathematical arguments are interesting in itself and highly important to the didactics of mathematics.

Description

Possible subjects are the parallellpostulate, ratio and real numbers, Cantor's diagonal argument, Cantor-Shroeder-Bernstein's thesis, the continuumhypothesis, the choice axioma, Banach and Tarski's paradox and infinite plays.
There will be some attention for the intuitionistical approach on these subjects.

Extra information

This course is meant for students in the E and C variant and other students who want to orientate in the foundations of mathematics. It will be given on request.

Galois Theory

Course id: **WM025B** *6 ec* Spring 2008 prof. dr. F.J. Keune

Teaching methods
- 28 hrs lecture
- 28 hrs tutorial

Learning outcomes
The student understands the role of group theory for the solution os algebraic equations in one unknown. He is able to formulate problems about algebraic equations in terms of field extensions.

Description
Automorphisms of field extensions. Normal and separable extensions. The main theorem of Galois theory. The Galois theory of cyclotomic extensions. Solubility of equations and groups. Solubility of special types of equations.

Literature
Lecture notes of the Institute WiNSt (dutch).

Examination
Written exam

Graph theory

Course id: **WM006A** *10 ec* dr. W. Bosma

Teaching methods
- 28 hrs lecture
- 28 hrs tutorial

Description
In its simplest form a graph is a set (of points/vertices) with a symmetric, non-reflexive relation, indicating whether two points are connected (by an edge) or not. Graphs are used e.g. to model networks for transport or communication as well as for assignment problems. In this course we will look at the fundamental concepts of graph theory, paying attention to combinatorial, algebraic, topological, but also to algorithmic aspects. Starting from elementary notions like distances, cycles, trees, connectedness or regularity we will proceed to coloured and labeled graphs which enlarge the field of possible applications. For example, in a transportation network the edges of a graph will be labeled by the costs arising on that connection.

Many natural questions on graphs give rise to interesting algorithmic methods, for example to find the distance (i.e. the shortest/cheapest path) between two points, to find the diameter of a graph (the largest distance between two points) or to determine short or long cycles in a graph. Furthermore, the connection of graph theory with other areas of algebra will be stressed, since many important graphs arise from groups, codes or designs.

Examination
To be consulted with lecturer

Group Theory

Course id: **NM028B** *9 ec* This course is given bi-annualy prof. dr. G.J. Heckman

Teaching methods
- 56 hrs lecture
- 28 hrs tutorial

Pre-requisites
Bachelor course 'Introduction to Group Theory' is a must

Learning outcomes
- The student acquires basic knowledge with the role of symmetry in quantum mechanics (e.g. Kramers degeneration and selection rules)
- The student is familiar with Wigner's Theorem on isomorphisms of ray space
- The student has familiarity with the concept of induced representations
- The student understands the unitarity spectrum of crystallographic groups and the Poincare group
- The student is familiar with the concept of universal enveloping algebra of a Lie algebra
- The student understands the classification of irreps of SU(2) and SU(3)
- The student has a clear understanding of the role of SU(2) and SU(3) for physics

Description
Group Theory plays a pivotal role in problems of mathematics and physics where one encounters symmetry of some sort. For example, molecular symmetry (if sufficiently rich) yields collision of spectral lines (spectral degeneration) and selection rules. Another example is the prominent role of the Lie group SO(3) in quantum mechanics, in particular in the exact treatment of the H-atom. These two applications have already been discussed in introductory courses in the bachelor study (Inleiding Groepentheorie and Inleiding Liegroepen).
The present course consists of three parts. The first topic is of a general nature about the use of groups in quantum mechanics (Wigner's theorem, Kramers degeneration, central extensions, Wigner-Eckhart theorem and selection rules).
The second part concerns induced representations with emphasis on the case of crystallographic groups (for solid state physics) and the Poincare group (for elementary particle physics).
Part three concerns Lie algebraic methods with special interest for the eightfold way Lie algebra SU (3) whose representation theory underlies the quark formalism in elementary particle physics.

Literature
Will be handed out during the course

Examination
Oral exam

Extra information
This course is given bi-annualy. Next one 2008-2009

Groups and representations

Course id: **WM010A** *10 ec* dr. B.D. Souvignier

Teaching methods
- 28 hrs lecture
- 28 hrs tutorial

Pre-requisites
Linear algebra, groups, rings and fields (Lineaire Algebra, Symmetrie, Ringen en Lichamen)

Learning outcomes
The student is acquainted with the basic theory of group representations and is able to deal with representations and characters in concrete examples. He knows how interesting properties of groups can be derived from their representations and characters and how the information required can be computed explicitly.

Description
In order to compute in an abstract group we require an explicit realization of the group elements. One possibility for such a realization is to view the group as a group of matrices defined as the image of a homomorphism from the group to a matrix ring. Such a homomorphism is called a representation of the group. The analysis of groups via their representations is a powerful tool which contributes to many and varied problems, such as the classification of finite simple groups, the theory of Lie groups or the determination of possible tilings of planes or spaces.

In this course we will both deal with the theory of group representations and with algorithmic methods that allow to apply the theory to explicit examples. In particular we will see how to construct representations, how to decompose representations into irreducible constituents and how to utilize the distillation of representations to their characters.

Literature
- M.Burrow: *Representation Theory of Finite Groups*, Academic Press, 1965
- J.H.Conway, R.T.Curtis, S.P.Norton, R.A.Parker, R.A.Wilson: *Atlas of Finite Groups*, Clarendon Press, 1985
- C.W.Curtis, I.Reiner: *Methods of Representation Theory*, Wiley-Interscience, 1981
- I.M.Isaacs: *Character Theory of Finite Groups*, Academic Press, 1976
- G.James, M.Liebeck: *Representations and Characters of Groups*, Cambridge UP., 1993
- A syllabus will be provided.

Examination
The students will be asked to hand in exercises and to present their solutions. The course will be rounded off by an oral examination.

Hilbert spaces and quantum mechanics

Course id: **WM053B** *6 ec* prof. dr. N.P. Landsman

Teaching methods
- 28 hrs lecture
- 28 hrs tutorial

Pre-requisites
Mathematics students: Analysis 1, 2, 3.
Physics students: Analysis 1 and Quantum mechanics 1 & 2.

Learning outcomes
The student is able to understand the abstract theory of Hilbert spaces and linear operators (bounded as well as unbounded) and can apply this theory to quantum mechanics.

Description
On one hand, this course gives an introduction to the mathematical theory of Hilbert spaces, which lie at the basis of modern analysis. On the other hand, quantum mechanics will be discussed in a mathematically rigorous way as a key application of the theory of Hilbert spaces.

Subjects
- Historical introduction
- Metric spaces, normed spaces, and Hilbert spaces
- Linear operators and functionals
- Compact operators
- Closed unbounded operators
- Spectral theory for closed unbounded operators
- Quantum mechanics and Hilbert space
- Spectral theory for selfadjoint operators
- Quantum logic

Literature
Lecture notes (preliminary version available from http://www.math.ru.nl/~landsman.HSQM.pdf)

Examination
Oral

Insurance Mathematics

Course id: **WM022B** *6 ec* dr. H.W.M. Hendriks

Teaching methods
- 28 hrs lecture
- 14 hrs tutorial

Pre-requisites
Basic course in Probability theory.

Learning outcomes
The student is familiar with utility functions and expected utility. He understands the individual and the collective risk model. He is familiar with Panjer's recursion. He understands the ruin theory according to Lundberg. He has insight in the properties of several risk premium principles. He understands the concept of reinsurance. The student is familiar with utility functions and expected utility. He understands the individual and the collective risk model. He is familiar with Panjer's recursion. He understands the ruin theory according to Lundberg. He has insight in the properties of several risk premium principles. He understands the concept of reinsurance.

Description
Utility functions, individual and collective risk model. Panjer's recursion. Cramér-Lundberg model for the surplus process, adjustment coefficient, Beekman's convolution formula for ruin probability. Various premium principles and their properties. Bonus-malus systems. First order and second order of stochastic dominance. In Insurance Mathematics methods are studied for determining premiums and for the management of the capital reserves, based on the data about the risks in a portfolio.

In this course we will discuss several premium calculation principles, mathematical models for the process of claiming times and claim sizes and bonus-malus systems. A highlight is the derivation of an integral equation for the probability of ruin (bankruptcy). That is the probability that, given a certain premium income and initial capital, at any moment the received premiums together with the initial capital are insufficient to cover the claims up to that moment. In the near future insurance companies will have to satisfy a continuity requirement, for which obviously the above theory could be applied. The course is meant for students interested in applications of mathematics in the financial world. It is part of the standard curriculum of Financial Mathematics.

Literature
- Kaas, R., *Modern actuarial risk theory*, Kluwer
- Bühlmann, H., *Mathematical methods in risk theory*, Springer

Examination
Written assignment with oral presentation.

Introduction to mathematical didactics

Course id: **WM043C** *6 ec* Fall dr. R.H. Kaenders

Teaching methods
- 28 hrs lecture
- 28 hrs tutorial

Learning outcomes
The student is familiar with didactical theories concerning the supervising of students in problem solving and independant research and uses these theories as a framework in reflection on that supervision.

Description
The introduction course to mathematical didactics is based on the course mathematical thinking, an activity organised by the Institute of Mathematics if the NWI faculty for pre-university education students. The Institute of Mathmatics offers these students the opportunity to create a profile paper about a mathematical subject, supervised by mathmaticians of the institute. The participants to the introduction course mathmatical didactics assist the mathematicians guiding de pre-university students within one of the offered theme's.

For an optimal supervision there are two conditions:
- Command of that part of mathematics in which students create their profile paper and a good view on favourable research questions for students.
- Particular didactics aimed at improvement of problem solving and support of research by students

Both aspects will be tought during the course.

Examination
There are two products on which the assessment will be based:
- An own mathematical investigation of the research area in which the students will create their profile paper, specially aimed at favourable research questions for students.
- A didactic reflection based on a protocol of an education fragment from a supervision conversation with students.

Introduction to Partial Differential Equations

Course id: **WB046B** *6 ec* Spring 2009 prof. dr. N.P. Landsman

Teaching methods
- 28 hrs lecture
- 28 hrs tutorial

Pre-requisites
This course is intended for third year or master students in mathematics and theoretical physics. It is compulsory for the master Mathematical Physics. Knowledge of the first half of Ordinary differential equationsis strongly recommended (though not absolutely necessary).

Learning outcomes
The student is able to solve simple partial differential equations and is able to make qualitative statements about equations that cannot be solved explicitly. The student knows the basic examples of both linear and nonlinear partial differential equations and has the ability to explore these examples.

Description
"In the mid-twentieth century the theory of partial differential equations was considered the summit of mathematics" (V.I. Arnold). " Our understanding of the fundamental processes of the natural world is based to a large extent on partial differential equations" (W.A. Strauss). Partial differential equations are clearly of fundamental importance inboth pure and applied mathematics, as well as in theoretical and mathematical physics. This course therefore offers an introductionin both the mathematical theory and the applications of partial differential equations.

Subjects
- First order equations
- Waves and Huygens' principle
- Fourier analysis of the vibrating string
- Oscillations and variational principle
- Harmonic functions
- The Laplacian
- Boundary value problems
- Nonlinear partial differential equations; Korteweg-de Vries equation

Literature
- Syllabus of Institute WiNSt
- V.I. Arnold, *Lecture on Partial Differential Equations* (Springer, 2004).
- Recommended: W.A. Strauss, *Partial Differential Equations: An Introduction* (Wiley, 1992).

Examination
Written exam

Intuitionistic mathematics

Course id: **WM037A** *10 ec* dr. W.H.M. Veldman

Teaching methods

- 28 hrs lecture
- 28 hrs tutorial

Learning outcomes

De student learns that mathematics may be developed in other ways than the usual one, in particular, along the lines indicated by the famous Dutch mathematician L.E.J. Brouwer.

Description

In this course we consider the criticism L.E.J. Brouwer (1881-1966) exercised on many results of classical real analysis, and explain why he refused to use the *principle of the excluded middle* in his own mathematical proofs. Brouwer not only wanted to restrict the logic of mathematical arguments but also proposed some new axioms. We will see that his new mathematics contains many delightful and convincing results.
We also treat *intuitionistic logic* as formalized by Heyting and Gentzen.
We will compare Brouwer's point of view with other conceptions of constructive mathematics.

Literature

- A. Heyting, *Intuitionism, an Introduction*, North Holland Publ. Co., Amsterdam 1971
- E. Bishop, D. Bridges, *Constructive Analysis*, Springer Verlag, New York etc.,1985
- D. Bridges, F. Richman, *Varieties of Constructive Mathematics*, Cambridge UP, 1987
- A.S. Troelstra. D. van Dalen, *Constructivism in Mathematics*, Volumes I and II, North Holland Publ. Co., 1988

Examination

After having completed and submitted a number of exercises, students have to pass an oral examination.

Knots and Groups

Course id: **WB034B** *6 ec* first semester dr. F.J.B.J. Clauwens

Teaching methods
- 28 hrs lecture
- 28 hrs tutorial

Pre-requisites
A fair part of the course Introduction to Topology and the course Symmetry (group theory)

Learning outcomes
The student is familiar with the concept of fundamental group. He is able to compute it in simple cases using the van Kampen theorem and the theory of covering maps. In particular he masters the necessary algebra e.g. calculations in groups using generators and relations. He knows about applications of the theory.

Description
- Winding number: definition, properties, computation, application.
- Fundamental group: definition and elementary properties.
- Free products of groups, descriptions of groups using generators and relations.
- The theorem of Van Kampen, with examples.
- The group of a link complement.
- Covering maps, lifts of maps, covering transformations.

The universal covering. Consider the sphere S={(x,y,z) in R^3, $x^2+y^2+z^2=1$} and the torus T={(x,y,z) in R^3, (-1+sqrt{x^2+y^2})^2+x^2=2}. Both spaces satisfy all the usual niceness conditions from Topology: they are compact, connected, Hausdorff etc. Thus it is difficult to show they are topologically different using only methods from Topology. To accomplish that we combine Topology with some Algebra, in particular the theory of groups. We undertake to give a precise meaning to the claim that "T has more holes than S" and to prove this claim subsequently. We analyze a general topolgical space X by considering closed curves in X up to deformation, and combining these into an abstract group, the so called fundamental group pi(X) of X. It will turn out that the fundamental group of S is trivial, whereas the fundamental group of T is free abelian of rank 2.

By way of introduction the course starts with the definition of the winding number w(c,p) which counts how many times a curve c in the planewinds around a point p in the plane. We develop methods to compute w easily. From the properties of w we prove a generalization of the Fundamental Theorem of Algebra, which says that for any polynomial of positive degree there is a complex number where it takes the value zero. Motivated by the properties of the winding number we define the fundamental group of an arbitrary space. We study the elementary properties of this concept: these suffice to show that S and T are indeed not homeomorphic.

A knot is a subspace of R^3 which is the continuous and injective image of a circle. We try to do distinguish knots by computing the fundamental groups of their complements. For this the elementary properties do not suffice and we need a bit more from the theory of groups than covered in the Symmetry course. If a space X is the union of two subspaces Y and Z meeting in a single point then its fundamental group pi(X) is a certain combination of their fundamental groups pi(Y) and pi(Z) called the free product. Thus part of the course is a digression about free products and related constructions in group theory. After this preparation we are even equipped to research smooth embeddings of finite graphs in 3-space. A famous example is that of the Borromean rings: three loops which cannot be separated without cutting,but after cutting any one of them the remaining two can be seperated.

A map f from a space X to a space Y is called a covering map if for each sufficiently small subset U of Y the inverse image f^(U) is a union of copies of U. An example is the exponential function from the complex numbers to the nonzero complex numbers. In the situation of a covering map there is a relation between the fundamental groups of X and Y. One can use this as a second method to compute fundamental groups, or as a way to apply prior knowledge about the fundamental groups. From the example given above it is obvious that mastering the theory of covering maps is very useful in studying complex functions and in particular in studying Riemann surfaces.

Literature
Lecture notes of the course

Examination
Oral

Lattices and crystallographic groups

Course id: **WM009A** *10 ec* dr. B.D. Souvignier

Teaching methods
- 28 hrs lecture
- 28 hrs tutorial

Pre-requisites
Linear algebra, groups, rings and fields.

Description
Crystallographic groups are the symmetry groups of lattices, i.e. of periodic discrete point sets in Euclidean space.They have applications in various fields, e.g. in solid state physics, but also give rise to esthetically appealing decorations.

In this course we will look at the basic notions of crystallographic groups from an algorithmic point of view. Concepts like lattices, point groups, group extensions and representations of groups will always be accompanied with algorithms to deal with them and with objects to which the algorithms can be applied, thus giving a hands-on experience.

As a consequence we will be able to determine the essentially different possibilities to decorate a frieze or a wall and the different forms of 3-dimensional crystals. The concepts which are introduced can be generalized to arbitrary dimensions and provide the basis to analyze more complicated structures like quasicrystals or bio-macromolecules.

Literature
A syllabus will be provided.

Lattices and Duality

Course id: **WB050B** *6 ec* fall 2008 prof.dr. M. Gehrke

Teaching methods
28 hours lecture
28 hours tutorial

Pre-requisites
Some mathematical maturity is the most important. Some knowledge of logic would be good, abstract algebra and/or the basics of general topology would be good even though I have in mind assuming none of this.

Learning outcomes
After this course the student is familiar with the notions of ordered algebraic structure, topo-relational structure, categorical duality and syntax and semantics of a propositional logic. She or he is familiar with several examples of propositional logics used in computer science such as intuitionistic logic and various forms of modal logic. He or she is familiarwith Stone/Priestley Duality for bounded distributive lattice-ordered algebras, and has seen several applications including one to knowledge representation.

Description
Algebra and geometry are, and have been since ancient times, the two main and separate strands of mathematics. Duality theory treats the part of algebra and topology (as a form of generalised geometry) where these strands are interchangeable and become one and the same. This setting turns out to be, essentially, the setting of propositional logic. In applications of logic we need to understand two aspects of a logic: the behavior of the system (its model theory) and the expressive/specification power of the system. These two fundamental aspects of logic are modelled by the two sides of topological duality.
The course will consist roughly of the following three segments:
- Partial orders, lattices, and their relation to logic. (8 lectures; 8 tutorials)
- Partial orders, topology, and their relationship to domain theory. (8 lectures; 8 tutorials)
- Duality theory for bounded distributive lattices with operators and relational semantics for propositional logics. (12 lectures; 12 tutorials)

Literature
- Parts of the book manuscript *'Lattices in Logic: canonicity, duality, and correspondence'* with H. A. Priestley

(Optional)
- B.A. Davey & H.A. Priestley, *Introduction to Lattices and Order*, 2nd edn (CUP 2002)
  S. Vickers, *Topology via Logic* (CUP 1989)
- P. Blackburn, M. de Rijke & Y. Venema, *Modal Logic* (CUP 2001)

Examination
Oral

Linear Programming

Course id: **WB044B** *6 ec* fall 2009

Teaching methods
- 28 hours lecture
- 28 hours tutorial

Learning outcomes
The student should get acquainted with the basic concepts of the field of linear programming. He should understand and be able to apply the basic theoretical results like the duality theorem and complementary slackness. Finally, he must understand the simplex method of Van Dantzig and be able to apply this algorithm in concrete situations.

Description
The course consists of a theoretical part and a practical part. Firstly, the course will cover the basic theorems in the theory of linear inequalities and the optimalization of linear functions over domains defined by linear inequalities (linear programming). The theory covers the duality theorem, the theory of complementary slackness and the decomposiion theorem of polyedral sets. As examples, the theorem of zero sum games and the core concept in transferable utility games will be discussed.

Machine Learning

Course id: **NM048B** *6 ec* first semester prof. dr. H.J. Kappen
dr. W.A.J.J. Wiegerinck

Website
http://www.snn.ru.nl/~wimw/collegeML.html

Teaching methods
- 40 hrs lecture

Pre-requisites
- Linear algebra, calculus, elementary probability theory (kansrekenen)

Learning outcomes
- The student is able to formulate a given stochastic multivariate system in terms of a probabilistic graphical model
- The student is able to formulate algorithms for exact and approximate inference in graphical models
- The student is able to formulate models and algorithms for (approximate) Bayesian machine learning

Description
This course provides an introduction to machine learning and neural networks from a probabilistic point of view. The ideas go back to the early days of cybernetics in the 1960s, when information theorists, computer scientists, physicists, mathematicians and neuroscientists all approached the problem to understand intelligence in biological systems from an engineering point of view. Nowadays, the probabilistic approach is used widely in modern machine learning, robotics, vision and artificial intelligence as well as in models of the brain.
The course gives a first introduction into this facinating field and containts the following topics: Probability, entropy and Bayesian inference; Graphical models; Clustering; Model comparison; Monte Carlo methods; Ising models; Independent component analysis; Variational methods; Neural networks, perceptrons; Associative memories; Gaussian processes.
The course is intended for senior bachelor students and master students in physics and mathematics. Computer science students are recommended to take the course 'Introduction to Pattern Recognition' prior to this course.

Literature
Necessary:
- David MacKay, *Information Theory, Inference and Learning Algorithms*, Cambridge University press. The entire book can be viewed on-screen at http://www.inference.phy.cam.ac.uk/mackay/itila/book.html

Other material will be distributed during the course.

Examination
Written exam

Model theory

Course id: **WM036A** *10 ec* dr. W.H.M. Veldman

Teaching methods
- 28 hrs lecture
- 28 hrs tutorial

Learning outcomes
The student becomes familiar with some results and techniques from model theory, the most important meeting point between mathematics and mathematical logic.

Description
In mathematics one often studies the class of structures satisfying a given set of formal axioms, for instance the class of groups, the class of fields, or the class of linear orders. In *Model theory* one starts to restrict oneself to the still rather general case that the axioms are formulated in a *first-order* or *elementary* language. This means that, when interpreting the formulas of such a language, one only quantifies over the domain of a given structure, and not, for instance, over the power set of the domain. One then asks questions like: given a structure, is it possible to axiomatize it, that is, is it possible to indicate a *not too difficult* set of formulas valid in the structure such that every formula valid in the structure logically follows from the formulas in the set. Or: given structures A, B, under what circumstances are A, B *elementarily equivalent*, that is, when do they satisfy the same elementary formulas? Or: given a set of formulas, how many countable structures do there exist satisfying all formulas in the set?
Model theory at its best is a delightful blend of abstract and concrete reasoning.

Literature
- C.C. Chang, H.J. Keisler, *Model Theory*, North Holland Publ. Co., Amsterdam, 1977
- G.E. Sacks, *Saturated Model Theory*, Benjamin, Reading, Mass.,1972
- B. Poizat, *Cours de théorie des Modèles*, Nur al-Matiq wal-Marífah, 1985
- W. Hodges, *A shorter Model Theory*, Cambridge University Press, 1997

Examination
After having completed and submitted a number of exercises, students have to pass an oral examination.

Modular forms

Course id: **WM030B** *8 ec* Not in 2007-2008 prof. dr. J.H.M. Steenbrink

Teaching methods
- 28 hrs lecture
- 28 hrs tutorial

Description
Modular forms are analytic functions on the complex upper half plane which have a very specific transformation behaviour under substitutions which are given by fractional linear transformations with integral coefficients. Modular forms are closely connected with elliptic curves. They are used to construct modular curves, i.e. spaces parametrizing isomorphism classes of elliptic curves with some extra structure, such as a rational point of given order. Wiles has recently shown that every 'stable'elliptic curve over the filed of rational numbers can be paramatrized by modular forms. This is a crucial step in his proof of Fermat's Last Theorem. The course will give an elementary introduction to modular forms; it presupposes some familiarity with elliptic curves and with complex function theory of one variable. It will be useful if some knowledge about Riemann surfaces is present. A text, based on an MRI Master Class course given in 2000-2001 can be found on my website. This course has a load of 8 ec. It can be chosen at any time. Typically, students study the subject themselves, and meet me once a week for about half an hour to ask questions.

Literature
- R.C. Gunning, *Lectures on modular forms*. Annals of Mathematics Studies 48, Princeton Univ. Press 1962
- A. Knapp, *Elliptic curves*. Mathematical Notes 40, Princeton Univ. Press 1992
- S. Lang, *Introduction to modular forms*. Springer 1976
- J.-P. Serre, *Cours d'arithmetique,* Presses Universitaires de France 1970
- G. Shimura, *Introduction to the arithmetic theory of Automorphic Functions*. Princeton Univ. Press 1971

Examination
Oral

Option Theory

Course id: **WM020B** *6 ec* prof. dr. M.C.A. van Zuijlen

Teaching methods

- 28 hrs lecture
- 28 hrs tutorial

Pre-requisites

The basic courses in Probability Theory and Statistics, Measure Theory, Stochastic Calculus. We remark that the course Stochastic Calculus can be followed simultaneously.

Learning outcomes

The student has a thorough understanding of the role of martingale methods and Itô theory in option theory in continuous time market models. Moreover, he is able to read and understand relevant papers in the literature on these topics.

Description

The standard call option offers the buyer the right but not the obligation to receive the stock for some strike price agreed in advance at a certain time in the future (the expiration date). For instance: a call option Philips, October, 2003, 20, gives the buyer the right to receive 100 stocks Philips at the third Friday in the month October 2003 for the price of 20 Euro per stock. Clearly, the holder of the contract will not use his right as long as the price of the stock Philips is below 20 at the mentioned date in October 2003. He will use his right and realise a gain (equal to the difference of the stock price and 20) if at that date the stock price of Philips is above Euro 20.

To buy such a call option one has to pay a premium (the option price or market price), which is determined on the option exchange and is varying through time, depending on the variation in the underlying stock price (Philips in the example). In their famous paper in 1973, Black and Scholes discovered on the basis of a stochastic differential equation a formula for the theoretical value of such an option. These theoretical values are computed nowadays by thousands on the exchanges in order to determine reasonable prices for options, to check the market prices of options and to develop investment strategies. Moreover the Black-Scholes formulas are used to measure and handle the risk of option portfolios.
Subjects: Finite security markets. Market inferfections. The Black-Scholes model. The limit transaction coming from the Cox-Ross-Rubenstein model to the Black-Scholes model. Foreign exchange market derivatives. American options.

Literature

- M. Baxter and A. Rennie, *Financial Calculus*. Cambridge University Press, 1996
  Thomas Björk, *Arbitrage theory in continuous time*, 2nd ed., Oxford University Press, 2004
- M. Musiela and M. Rutkowski, *Martingale methods in financial modeling*. Springer Verlag, Berlin Heidelberg, 2005

Examination

Written assignment with oral presentations.

Polynomial mappings

Course id: **WM027A** *10 ec* Spring dr. A.R.P. van den Essen

Teaching methods
- 28 hrs lecture
- 28 hrs tutorial

Pre-requisites
Commutative Algebra

Learning outcomes
The student is familiar with the theory of derivations and its applications to the study of polynomial automorphisms. He has an overview of the various main problems in the field.

Description
A polynomial mapping from $\mathbf{C}^n$ to itself is a mapping whose components are polynomials in n variables with complex coefficients. A linear mapping is an example, namely in this case each component is a linear polynomial in the variables. Linear mappings play a fundamental role when one wants to solve systems of linear equations. The following results were obtained in the linear algebra courses. A linear mapping is invertible if the determinant of the corresponding matrix is invertible in $\mathbf{C}$. If a linear mapping from $\mathbf{C}^n$ to itself is injective, then it is invertible and the inverse is again a linear mapping. Every invertible matrix is a product of elementary matrices. In this course we investigate how (if possible) these results can be generalised to polynomial mappings. The research on this kind of questions has led to interesting new results and relations with problems in other areas of mathematics(such as dynamical systems and cryptography) but also it gave rise to various famous conjectures and unsolved problems. These attractive problems, such as the Jacobian conjecture which is an attempt to generalise the first item above, have made the field of polynomial mappings into a rapidly growing research area. We discuss the basic results of the theory as well as several of the very recent new discoveries which take place at the forefront of research in this exciting part of Mathematics.

Literature
We study parts from the book *'Polynomial Automorphisms and the Jacobian Conjecture'*, by Arno van den Essen, Vol. 190 in Progress in Mathematics, Birkhauser (2000).

Examination
The student can make a series of exercises or do research in a topic and write a report on it.

Portfolio management

Course id: **WM023B** *6 ec* Fall 2007 dr. H.W.M. Hendriks

Teaching methods
- 28 hrs lecture
- 28 hrs tutorial

Pre-requisites
Stochastic Calculus, Option Theory

Learning outcomes
The ultimate goal is to open, for the student, the literature on portfolio management.

Description
Introduction to Markowitz approach. Portfolio management with the possibility to trade with a finite number of stock (and one bond). Realization of terminal wealth in a complete market. Utility functions. Optimizing consumption and terminal wealth using stochastic control approach and using the martingale approach. Hamilton-Jacobi-Bellman equation. Verification theorems.

Portfolio Management encompasses in its ultimate form all aspects of financial management. We will expound the Markowitz model which is a quantitative description in which the interplay between (expected) yield and risk is made clear. We will study portfolio management for a finite period and in continuous time. Important things are the consumption process and the final value of the portfolio. In the context where the financial tools are a bank account and a stock, whose value process is geometric Brownian motion, the needed initial capital to realize a given consumption and final value is within grasp of ordinary option theory. If, given suitable utility functions, one tries to optimize the total expected utility, interesting techniques have to be introduced. We will discuss the Hamilton-Jacobi-Bellman approach and a martingale approach.

Literature
Korn, R., *Optimal Portfolios*, World Scientific.

Examination
Written assignment with oral presentation.

Probability Theory

Course id: **WB022B** *6 ec* second semester dr. H.W.M. Hendriks

Teaching methods
- 28 hrs lecture
- 28 hrs tutorial

Pre-requisites
The introductory courses Probability, Advanced Probability, Introduction in Statistics and the course Measure and Integral.

Learning outcomes
The student has mastered the translation between measure and integral theoretical notions and probabilistic notions. Understanding and skill in the different notions of convergence, and the notion of independence of families of sigma-algebras. Understanding and skill in conditional expectation and familiarity with martingales.

Description
The Kolmogorov axiomatics of probability theory. Expected value, moments, Jensen inequality. Types of convergence. Independence of sigma-algebras. Zero-One Laws, Laws of large numbers, Central limit theorem. Conditional expectation with respect to a sub-sigma-algebra. Martingales in discrete time and basic properties.

In this course we treat the foundation of modern probability theory as an application of measure and integration theory. The central theorems, like the lows of large numbers and the central limit theorem are treated in great generality.

Moreover the notions of conditional expectation and conditional probability will be broadened. We will pay attention to martingales, which are an important ingredient in the description of financial price processes.

Literature
- Lecture notes of the Mathematical Institute
- Durrett, R., Probability: Theory and Examples, Duxbury

Examination
Oral

Stochastic calculus

Course id: **WM019B** *6 ec* Fall dr. J.D.M. Maassen

Teaching methods
- 28 hrs lecture
- 28 hrs tutorial

Pre-requisites
Measure and integral, Probability theory

Learning outcomes
The student is able to work out solutions of diffusion equations by Ito calculus, and to determinate their properties.

Description
In the 1950's K. Itô developed a differential calculus to deal with random functions. This fine piece of functional analysis has found application in a wide range of fields, extending from electronic engineering via quantum field theory and population biology to the stock market. In this course Itô diffusions are constructed, their calculus is developed, and several applications are treated. The course will be taught if sufficiently many participants show up.

Subjects
- Brownian motion
- The Itô integral
- Stochastic differential equations
- Applications to potential theory
- Path integrals
- The Kalman filter
- The Black and Scholes option price

Literature
Lecture notes.

Examination
Oral exam

The arithmetic of elliptic curves

Course id: **WM028B** *6 ec* Not in 2007-2008 prof. dr. J.H.M. Steenbrink

Teaching methods
- 28 hrs lecture
- 28 hrs tutorial

Pre-requisites
Analysis and algebra of the first two years.

Description
This course deals with the geometry and arithmetic of solutions of equations of the form $y^2 = f(x)$ where $f(x)$ is a squarefree monic cubic polynomial with rational coefficients. If one adds one point at infinity, these solutions form a finitely generated abelian group (the Mordell-Weil theorem). This will be proved in the case where f has a rational zero. These elliptic curves are closely related to several other problems in algebraic number theory, such as the congruent number problem (a rational number is 'congruent' if it is the area of a right triangle with all sides rational; the problem is to give a method to determine whether a given integer is a congruent number).

Literature
- A. Knapp, Elliptic curves. *Mathematical Notes 40*, Princeton Univ. Press 1992
- Joseph H. Silverman, John Tate: *Rational points on elliptic curves. Undergraduate Texts in Mathematics*. Springer Verlag 1992

Examination
Oral

The Structure of Spacetime

Course id: **WM058B** *6 ec* second semester dr. W.D. van Suijlekom
dr. E.J. Hawkins

Teaching methods

- 28 hrs lecture
- 28 hrs tutorial

Pre-requisites

'Tensoren en Toepassingen' and 'Inleiding Algemene Relativiteitstheorie'

Learning outcomes

The student has a conceptual understanding of the mathematical structure of General Relativity and is able to understand the research literature on gravitational physics.

Description

We introduce the mathematical techniques necessary for applying Einstein's general theory of relativity. These include the concepts of manifolds, curvature, symmetries, differential forms, and conformal/causal structure. Using these, we will cover singularity theorems, integral theorems, and applications to cosmology and the death of stars.

Literature

R.M. Wald. *General Relativity.* University of Chicago Press, 1984. (ISBN 9780226870335)

Theory of recursive functions: computability, unsolvability and unprovability

Course id: **WM039A** *10 ec* Fall 2007 dr. W.H.M. Veldman

Teaching methods
- 28 hrs lecture
- 28 hrs tutorial

Learning outcomes
The student learns the notion of a computable function, that is of central importance both for mathematics and for theoretical computer science.

Description
We describe various approaches of the class of partial computable functions from *N* to *N*, due to Turing, Kleene, Markov and others and prove their equivalence. We consider subclasses of this class, for instance the class of *elementary functions* and the class of *primitive recursive functions*. We study *acceptable sets*, Kleene's *Recursion Theorems*, and the *Kleen-Mostowski-hierarchy*. We consider *Post's problem* and its solution by Friedberg and Mucnik. Finally, we consider Gödel's *Incompleteness Theorems.*

Literature
- S.C. Kleene, *Introduction to Metamathematics*, North Holland Publ. Co., Amsterdam 1952
- H. Rogers, *Theory of Recursive Functions and Effective Computability*, Mc Graw Hill, 1967
- G. Boolos, R. Jeffrey, *Computability and Logic*, Cambridge UP, 1974
- N.J. Cutland, Computability, *an Introduction to Recursive Function Theory,* Cambridge University Press, 1980
- P.G. Odifreddi, *Classical Recursion Theory*, Vol. I and II, Elsevier, Amsterdam, 1989, 1999

Examination
After having completed and submitted a number of exercises, students have to pass an oral examination.

Undergraduate algebraic geometry

Course id: **WM031B** *6 ec* Not in 2007-2008 prof. dr. J.H.M. Steenbrink

Teaching methods
- 28 hrs lecture
- 28 hrs tutorial

Description
This is an experimental course, based on reading the book 'Undergraduate Algebraic Geometry'by Miles Reid. It deals with polynomial equations in two and three variables (and a bit of general theory) focussing on conics (the case of degree two) and cubic curves and surfaces. By studying these particular cases and developing some tools to study them you may acquire a bit of the flavour of this beautiful subject!

Literature
'Undergraduate Algebraic Geometry' by Miles Reid, London Mathematical Society Student Texts 12, Cambridge University Press 1988

Examination
Oral

5.1 List of lecturers

Bosma, Dr. W.	w.bosma@math.ru.nl	52311	HG03.716
Clauwens, Dr. F.J.B.J.	f.clauwens@math.ru.nl	52996	HG03.713
Essen, Dr. A.R.P. van den	a.vandenessen@math.ru.nl	52993	HG03.715
Gehrke, Prof.dr. M.	M.Gehrke@math.ru.nl		
Hawkins, Dr. E.J.	e.hawkins@math.ru.nl		
Heckman, Prof. dr. G.J.	g.heckman@science.ru.nl	53233	HG03.077
Hendriks, Dr. H.W.M.	h.hendriks@science.ru.nl	52868	HG03.720
Kaenders, Dr. R.H.	r.kaenders@science.ru.nl	30090	E20.22
Kappen, Prof. dr. H.J.	b.kappen@science.ru.nl	14241	
Keune, Prof. dr. F.J.	f.keune@science.ru.nl	53230	HG03.718
Landsman, Prof. dr. N.P.	landsman@math.ru.nl	52874	HG03.078
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